MATH 202.14 (Term 172)

Quiz 3 (Sects. 4.1, 4.2 & 4.3) Duration: 20min

Name: ID number:

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = e^x$ and $Ly_{p_2} = e^{-x}$, then find a particular solution of the DE $Ly = \cosh x + 2\sinh x$.

2.)(3pts) Given that $y_1 = (x+1)^2$ is a solution to the DE $(x+1)^2 y'' - 2y = 0$, find a second solution of the DE that is linearly independent to y_1.

3.)(4pts) Solve the DE: $y''' - 4y'' + y' - 4y = 0$.

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1) $\cosh x = \frac{e^x + e^{-x}}{2}$, $\sinh x = \frac{e^x - e^{-x}}{2}$

$\Rightarrow \cosh x + 2\sinh x = \frac{3}{2}e^x - \frac{e^{-x}}{2}$

$\Rightarrow y_p = \frac{3}{2}y_{p_1} - \frac{1}{2}y_{p_2}$

2.) $(x+1)^2 y'' - 2y = 0$

$y'' + 0y' - \frac{2}{(x+1)^2}y = 0$

Using reduction of order formula, we find

$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\,dx$

$P = 0 \Rightarrow e^{-\int P(x)dx} = 1$

$\Rightarrow y_2 = (x+1)^2 \int \frac{1}{(1+x)^4}\,dx$

$= (x+1)^2 \left(\frac{-1}{3}\right)\frac{1}{(x+1)^3} = -\frac{1}{3}\frac{1}{x+1}$

$\boxed{y_2 = \frac{1}{x+1}}$

3.) Auxiliary equation is

$m^3 - 4m^2 + m - 4 = 0$

$m^2(m-4) + m - 4 = 0$

$(m-4)(m^2+1) = 0$

$m = 4$, $m^2 = -1$

$m = \pm i$

$\Rightarrow y = c_1 e^{4x} + c_2 \cos x + c_3 \sin x$,

where c_1, c_2, c_3 are arbitrary constants.