MATH 202.14 (Term 172)
Quiz 1 (Sects. 2.2 & 2.3) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the IVP: $\begin{cases} y^2\frac{dy}{dx} = (y-2)(y+1)\cos^5 x \sin x \\ y(0) = 1. \end{cases}$

2.)(5pts)Solve the DE: $(x-1)\frac{dy}{dx} + y = \frac{(x-1)}{x^2+1}$.

1.) Constant solutions are $y=2$ and $y=-1$

They are rejected because $y(0) \neq 1$ for both

Now, assume $y \neq 2, y \neq -1$

$$\int \frac{y^2}{(y+1)(y-2)}\,dy = \int \cos^5 x \sin x\,dx$$

$$\frac{y^2}{(y+1)(y-2)} = 1 + \frac{a}{y+1} + \frac{b}{y-2}$$

$a = -\frac{1}{3}$ and $b = \frac{4}{3}$

$$\int\left(1 - \frac{\frac{1}{3}}{y+1} + \frac{\frac{4}{3}}{y-2}\right)dy = -\frac{\cos^6 x}{6} + C$$

$$y - \frac{1}{3}\ln|y+1| + \frac{4}{3}\ln|y-2| = -\frac{\cos^6 x}{6} + C$$

$$3y + \ln\left|\frac{(y-2)^4}{y+1}\right| = -\frac{\cos^6 x}{2} + C$$

$$\frac{(y-2)^4}{y+1} = c\,e^{-3y - \frac{\cos^6 x}{2}}$$

$y(0)=1 \Rightarrow c = \frac{1}{2}e^{7/2}$

$$\Rightarrow \frac{(y-2)^4}{y+1} = \frac{1}{2}e^{-3y - \frac{\cos^6 x}{2} + \frac{7}{2}}, \quad x \in I$$

2.) $\frac{dy}{dx} + \frac{y}{x-1} = \frac{1}{x^2+1}$

$$e^{\int \frac{dx}{x-1}} = e^{\ln|x-1|} = x-1, \quad x > 1$$

$$\Rightarrow \frac{d}{dx}\big(y(x-1)\big) = \frac{x-1}{x^2+1}$$

$$y(x-1) = \int \frac{x-1}{x^2+1}\,dx$$

$$= \int\left(\frac{1}{2}\frac{2x}{x^2+1} - \frac{1}{x^2+1}\right)dx$$

$$y(x-1) = \frac{1}{2}\ln(x^2+1) - \tan^{-1}x + C$$

$$\Rightarrow y = \frac{1}{x-1}\left[\frac{\ln(x^2+1)}{2} - \tan^{-1}x + C\right],$$

for $x > 1$