

Name:

KFUPM ID:

**Exercise 1**

Let  $L$  be a linear differential operator such that

$$L(e^x) = x^2 + 1, \quad \text{and} \quad L(xe^x) = x^3 + 3$$

Find a particular solution of the differential equation

$$L(y) = \frac{x^2}{2} \left(1 - \frac{x}{3}\right).$$

**Exercise 2**

1. Show that  $y_1 = e^x$  and  $y_2 = xe^{x^2}$  are linearly independent functions on  $\mathbb{R}$ .
2. Show that  $f_1 = \cos(3x)$  and  $f_2 = \cos^2(3x/2), f_3 = 1$  are linearly dependent functions on  $\mathbb{R}$ .

**Exercise 3**

Solve the differential equation

$$x^2 y'' - (2x^2 + x)y' + (x^2 + x)y = 0,$$

given that  $y_1 = e^x$  is a solution.

**Exercise 4**

Solve the differential equation

$$(D^3 - 7D^2 + 16D - 12)y = 0,$$

given that  $y_1 = e^{2x}$  is a solution.