# Additional Exercises

## Math 202: Sections 3.1


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**This is a list of exercises with corrections. We do not pretend that this list covers entirely the whole material of the indicated sections, and we warmly recommend to not stop practicing after finishing these exercises. Never forget that knowledge is a treasure, but practice is the key to it.**

## 1 Statement

### Exercise I

An object with Temperature 108 °F was placed outside where the temperature was 20°F. On Sunday, January 01, 2017 at 00:00, the temperature was 60°F and at 00:02, its temperature was 30°F. At what date and time was the object placed outside.

### Exercise II

A population of birds is known to increase at a rate proportional to the number of birds present at time t. If an initial population $P_0$ has doubled in 5 years, how long will it takes to triple ?

### Exercise III

A population of rats with predators is known to decrease at a rate proportional to the population present at time t. If $20\%$ of the original population $P_0$ disappears in $70$ days, then find the number of rats present after 140 days. If the initial population is $P_0 = 100$, estimate the number of days necessary within which the population will disappear.

### Exercise IV

A Thermometer reading 70 °F is placed in an oven preheated to a constant temperature. Though the oven's glass, an observer records that the thermometer reads 110 °F after $1/2$ minute and 145°F after one minute. What is the constant temperature of the oven ?

## 2 Correction

### Exercise I

Let $\theta(t)$ be the temperature of the object at time $t$. Then, by Newton's Law of Cooling, we have

$$\theta'(t) = k(\theta(t) - A). \tag{1}$$

here $A = -20$ which is the surrounding temperature with respect to the object. We let $00:00$ be the origin of time and $mn$ its unit. Then, we have $\theta(0) = 60$ and $\theta(2) = 30$. The solution of the DE (1) is given by

$$\theta(t) = A + c\,e^{kt} = -20 + c\,e^{kt},$$

and

$$\theta(0) = 60 \iff 60 = -20 + c \iff c = 80.$$

Also

$$\theta(2) = 30 \iff 30 = -20 + 80\,e^{2k} \iff k = \frac{1}{2}\ln\frac{5}{8}.$$

Thus

$$\theta(t) = -20 + 80\,e^{\frac{1}{2}\ln\frac{5}{8}\,t}.$$

Now, let us assume that the object was placed out at time $T$. Thus $\theta(T) = 108$. This means

$$108 = -20 + 80\,e^{\frac{1}{2}\ln\frac{5}{8}\,T} \iff T = -2.$$

### Exercise II

Let $P(t)$ be the number of birds of the population at time $t$. Then the model governing the behaviour of the number of the population is

$$P'(t) = k\,P(t), \quad P(0) = P_0, \quad P(5) = 2P(0). \tag{2}$$

The solution is clearly given by

$$P(t) = P_o\,e^{kt}.$$

As $P(5) = 2P(0)$, we get

$$P_0\,e^{5k} = 2P_0 \quad\iff\quad k = \frac{1}{5}\ln 2.$$

Thus,

$$P(t) = P_0\, e^{\frac{1}{5}\ \ln 2\, t}$$

Eventually, let us denote the required time for the population to triple by $T$. Therefore

$$P(T) = 3P_0 \quad \Longleftrightarrow \quad 3P_0 = P_0 e^{\frac{1}{5}\ \ln 2\, T}.$$

It is rather easy to see that

$$T = 5\frac{\ln 3}{\ln 2}.$$

**Exercise III**

Let $P(t)$ be the number of rats at the time $t$. The evolution of this number is modeled by the DE

$$P'(t) = k\, P(t).$$

The solution of this DE is given by

$$P(t) = P_0\, e^{kt}.$$

Since $P(70) = \frac{80}{100}\, P_0 = \frac{4}{5}\, P_0$, we deduce that

$$\frac{4}{5}\, P_0 = P_0\, e^{70k} \quad \Longleftrightarrow \quad k = \frac{1}{70}\, \ln \frac{4}{5}.$$

Thus, we have

$$P(t) = P_0\, e^{\frac{1}{70}\ \ln \frac{4}{5}\ t}.$$

Consequently

$$P(140) = \frac{16}{25}\, P_0.$$

Let us assume that the population will disappear at time $T$ corresponding to $P(T) < 1$. That is

$$\begin{aligned}
P(T) < 1 \Longleftrightarrow \quad & P_0\, e^{\frac{1}{70}\ \ln \frac{4}{5}\ T} < 1 \\
\Longleftrightarrow \quad & \frac{1}{70}\, \ln \frac{4}{5}\, T < \ln \frac{1}{P_0} = -\ln P_0 \\
\Longleftrightarrow \quad & \frac{1}{70}\, \ln \frac{5}{4}\, T > \ln P_0 \\
\Longleftrightarrow \quad & T > 70\, \frac{\ln P_0}{\ln 5 - \ln 4} \\
\Longleftrightarrow \quad & T > 70\, \frac{\ln 100}{\ln 5 - \ln 4} \simeq 1444.63.
\end{aligned}$$

Thus, the population will disappear after $1444.63$ days.

**Exercise IV**

Let $A$ be the constant temperature of the oven. We let the zero-time be the time when the thermometer was placed in the oven. Let $\theta(t)$ be the temperature read on the thermometer at time $t$. This model is governed by the DE

$$\theta'(t) = k(\theta(t) - A).$$

Clearly, its solution is given by

$$\theta'(t) = A + c\, e^{kt}, \quad c \in \mathbb{R}.$$

Since $\theta(0) = 70$ and $\theta(\frac{1}{2}) = 110$, we have $70 = A + c$

$$A + c = 70, \quad \text{and} \quad 110 = A + c\, e^{\frac{k}{2}}$$

respectively. Also, we have $\theta(1) = 145$, that is

$$145 = A + c\, e^{k}.$$

Thus, we have the fofflowing set of equations

$$\begin{cases} c = 70 - A, \\ 110 = A + (70 - A)\, e^{\frac{k}{2}}, \\ 145 = A + (70 - A)\, e^{k}. \end{cases}$$

The second equation leads to

$$(110 - A)^2 = (70 - A)^2\, e^{k},$$

Using teh third equation, we get

$$(110 - A)^2 = (70 - A)^2\, e^{k} = (70 - A)(145 - A).$$

Therefore

$$110^2 - 220A + A^2 = 70 \times 145 - 215A + A^2.$$

Solving this equation leads to

$$A = 390.$$

Therefore, the constant preheating temperature of the oven is $390°$ F.