Name:

KFUPM ID:

Exercise 1

Let L be a linear differential operator such that

$$L(e^x) = x^2 + 1, \quad \text{and} \quad L(xe^x) = x^3 + 3$$

Find a particular solution of the differential equation

$$L(y) = \frac{x^2}{2}\left(1 - \frac{x}{3}\right).$$

Exercise 2

1. Show that $y_1 = e^x$ and $y_2 = xe^{x^2}$ are linearly independent functions on $\mathbb{R}$.

2. Show that $f_1 = \cos(3x)$ and $f_2 = \cos^2(3x/2), f_3 = 1$ are linearly dependent functions on $\mathbb{R}$.

Exercise 3

Solve the differential equation

$$x^2 y'' - (2x^2 + x) y' + (x^2 + x) y = 0,$$

given that $y_1 = e^x$ is a solution.

Exercise 4

Solve the differential equation

$$(D^3 - 7D^2 + 16D - 12)\,y = 0,$$

given that $y_1 = e^{2x}$ is a solution.