Name:

KFUPM ID:

Exercise 1

Consider the following differential equation

$$y' - y^2 + 3y - 2 = 0. \tag{1}$$

1. Show that
$$y(x) = \frac{2 - c\,e^x}{1 - c\,e^x},$$
is a one-parameter family of solutions of the differential equation (1).
2. Find a singular solution of the DE (1).

Exercise 2

Consider the following initial value problem

$$\begin{cases} y' = \sqrt{y^2-4} + \sqrt{9-x^2}, \\ \\ y(x_0) = y_0. \end{cases} \tag{2}$$

1. Find and sketch the region of all $(x_0, y_0) \in \mathbb{R}^2$ such that the initial value problem (2) has a unique solution following the Theorem of existence uniqueness of solutions.
2. Find the largest interval on which the solution of the IVP (2) with $x_0 = 1$ and $y_0 = 3$ may be defined.

Exercise 3

Solve the following initial value problem

$$\begin{cases} y' - x\,e^{x+y} = 0, \\ \\ y(0) = 0, \end{cases}$$

and find the largest interval on which the solution may be defined.

Exercise 4

1. Evaluate the derivative of the function $1 + (\ln x)^2$.
2. Solve the following differential equation

$$x\left(1 + (\ln x)^2\right) y' + 2\ln(x)\, y = 1,$$

on $(0, +\infty)$.