Math201.01, Quiz #2, Term 172

Name:

Solutions

ID#: **Serial #:**

1. **[3 points]** Find an equation of the plane that contains the point $(1,0,-2)$ and the line $x = 3t, y = 1 + t, z = 2 - t$.
2. **[3 points]** Identify (name, vertex, axis) and sketch the surface $x^2 - 2x + 2y^2 + 8y - z = -12$.
3. **[2 points]** Find and sketch the domain of $f(x,y) = \frac{\ln(x-1)}{y-x^2}$.
4. **[2 points]** Find the limit if it exists: $\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$.

Good luck,

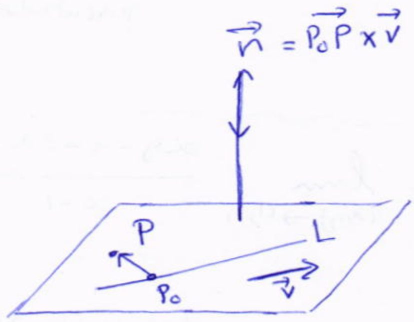
Ibrahim Al-Rasasi

$\boxed{1}$. $P(1,0,-2)$ a point in the plane.

. $t=0 \Rightarrow P_0(0,1,2)$ 0.5

. a vector parallel to the line : $\vec{V} = \langle 3,1,-1 \rangle$. 0.5

. The normal vector to the plane is



0.5 $\vec{n} = \overrightarrow{P_0P} \times \vec{v}$, $\overrightarrow{P_0P} = \langle 1,-1,-4 \rangle$ 0.5

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -4 \\ 3 & 1 & -1 \end{vmatrix}$$

0.5 $= 5\vec{i} - 11\vec{j} + 4\vec{k}$

. an equation of the plane is

$$5(x-1) - 11(y-0) + 4(z+2) = 0$$

$$5x - 11y + 4z - 5 + 8 = 0$$

$$5x - 11y + 4z = -3$$ 0.5

2 $x^2-2x+2y^2+8y-z=-12$

$x^2-2x+1+2(y^2+4y+4)=z-12+1+8$

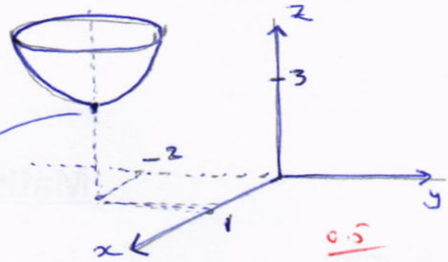
$(x-1)^2+2(y+2)^2=z-3$

or $z=(x-1)^2+2(y+2)^2+3$ 1

0.5 . an elliptic paraboloid

0.5 . vertex : $(1,-2,3)$

0.5 . axis : the line through the vertex $(1,-2,3)$ and parallel to the z-axis.



0.5

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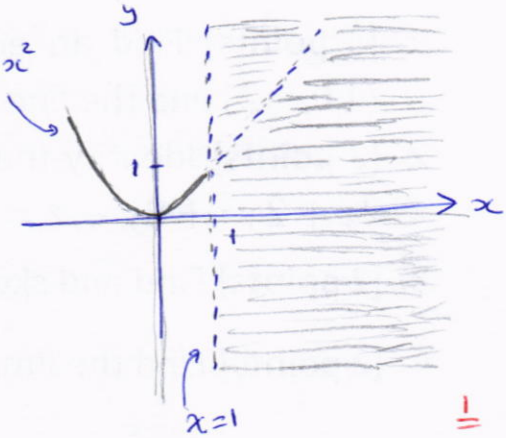
3 $f(x,y)=\dfrac{\ln(x-1)}{y-x^2}$

Domain $=\{(x,y):\ x-1>0 \text{ and } y-x^2\neq 0\}$

$=\{(x,y):\ x>1 \text{ and } y\neq x^2\}$ 1

= all points (x,y) to the right of the line $x=1$ and not on the parabola $y=x^2$.



1

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4 $\displaystyle\lim_{(x,y)\to(1,1)}\frac{xy-y-2x+2}{x-1}$, $\frac{0}{0}$, undefined

Factor : $xy-y-2x+2$

$=y(x-1)-2(x-1)$

$=(x-1)(y-2)$ 1

$=\displaystyle\lim_{(x,y)\to(1,1)}\frac{(x-1)(y-2)}{x-1}$

$=\displaystyle\lim_{(x,y)\to(1,1)} y-2$ 0.5

$= 1-2=-1.$ 0.5

③

Math201.02, Quiz #2, Term 172

Name:

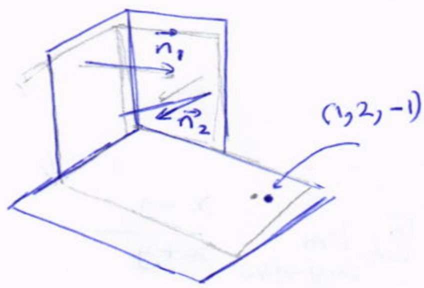
Solutions

ID#: **Serial #:**

1. **[3 points]** Find an equation of the plane that passes through the point $(1,2,-1)$ and is perpendicular to the planes $2x+y-2z=2\ and\ x+3z=4$.
2. **[3 points]** Identify (name, vertex, axis) and sketch the surface $2x^2+4x+y^2-2y-z^2=-3$.
3. **[2 points]** Find and sketch the domain of $f(x,y)=\sqrt{xy-1}$.
4. **[2 points]** Find the limit if it exists: $\lim_{(x,y)\to(0,0)}\frac{x-y}{x+y}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$ $P_1: 2x+y-2z=2 \implies \vec{n}_1=\langle 2,1,-2\rangle$ 0.5

$P_2: x+3z=4 \implies \vec{n}_2=\langle 1,0,3\rangle$ 0.5

- a normal vector to the required plane is 1

$$\vec{n}=\vec{n}_1\times\vec{n}_2 = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 1 & -2\\ 1 & 0 & 3\end{vmatrix}$$

$$= 3\vec{i}-8\vec{j}-\vec{k} \stackrel{\text{or}}{=} \langle 3,-8,-1\rangle$$ 0.5

- an equation of the plane is

$$3(x-1)-8(y-2)-(z+1)=0$$

$$\implies 3x-3-8y+16-z-1=0$$

$$\implies 3x-8y-z=-12$$ 0.5

(4)

2 $2x^2+4x+y^2-2y-z^2=-3$

$2(x^2+2x+1)+(y^2-2y+1)=z^2-3+2+1$

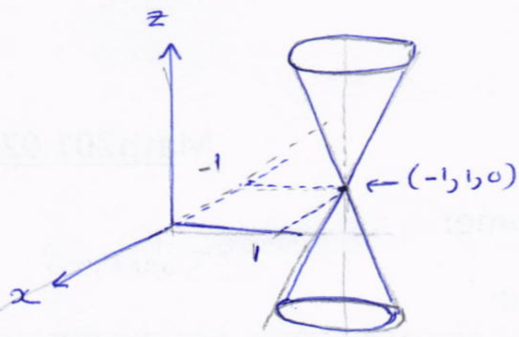
$2(x+1)^2+(y-1)^2=z^2$ 1

0.5 • an elliptic cone

0.5 • vertex : $(-1,1,0)$

0.5 • axis : the line through the vertex $(-1,1,0)$ and is parallel to the z-axis

0.5



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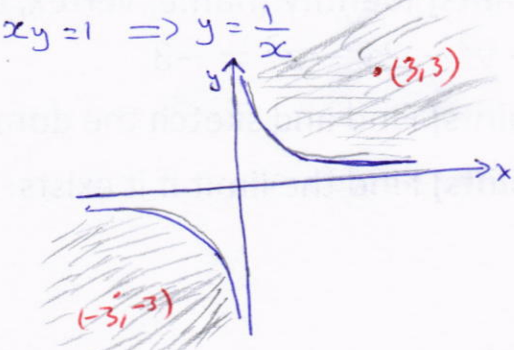
3 $F(x,y)=\sqrt{xy-1}$

Domain $=\{(x,y): xy-1\geq 0\}$

$=\{(x,y): xy\geq 1\}$

1

$xy=1 \Rightarrow y=\frac{1}{x}$



1

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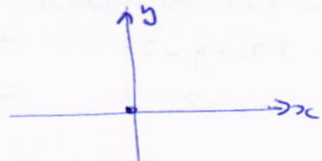
4 $\lim_{(x,y)\to(0,0)}\frac{x-y}{x+y}$



• along the x-axis : $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}}\frac{x-y}{x+y}=\lim_{(x,y)\to(0,0)}\left.\frac{x-y}{x+y}\right|_{y=0}=\lim_{(x,y)\to(0,0)}\frac{x-0}{x+0}=\lim_{(x,y)\to(0,0)}1=1$$

• along the y-axis : $x=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}}\frac{x-y}{x+y}=\lim_{(x,y)\to(0,0)}\left.\frac{x-y}{x+y}\right|_{x=0}=\lim_{(x,y)\to(0,0)}\frac{0-y}{0+y}=\lim_{(x,y)\to(0,0)}-1=-1$$

Since the limits along the two paths are not equal, then

$$\lim_{(x,y)\to(0,0)}\frac{x-y}{x+y}\ \text{DNE}.$$

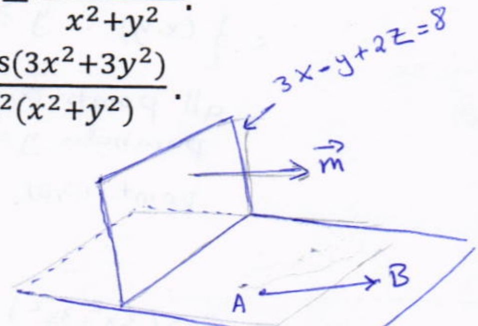
Math201.04, Quiz #2, Term 172

Name:

Solutions

ID#: **Serial #:**

1. **[3 points]** Find an equation of the plane that contains the points $A(0,2,1)$ *and* $B(-1,3,1)$ and perpendicular to the plane $3x - y + 2z = 8$.
2. **[3 points]** Identify (name, vertex(ces), axis) and sketch the surface $z^2 - 19 = x^2 + 2y^2 - 12y$.
3. **[2 points]** Find and sketch the domain of $f(x,y) = \frac{\sqrt{y-x^2+1}}{x^2+y^2}$.
4. **[2 points]** Find the limit if it exists: $\lim\limits_{(x,y)\to(0,0)} \frac{1-\cos(3x^2+3y^2)}{\sin^2(x^2+y^2)}$.



Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ • $\overrightarrow{AB} = \langle -1-0, 3-2, 1-1 \rangle = \langle -1, 1, 0 \rangle$ 0.5

• $3x - y + 2z = 8 \implies \vec{m} = \langle 3, -1, 2 \rangle$ 0.5

• a normal vector to the required plane is 1

$$\vec{n} = \overrightarrow{AB} \times \vec{m} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 2\vec{i} + 2\vec{j} - 2\vec{k} \overset{\text{or}}{=} \langle 2, 2, -2 \rangle$$ 0.5

, an equation of the plane is (using $\vec{n}$ and $A(0,2,1)$):

$$2(x-0) + 2(y-2) - 2(z-1) = 0 \qquad (\div 2)$$

$$\implies x + y - 2 - z + 1 = 0$$

$$\implies x + y - z = 1$$ 0.5

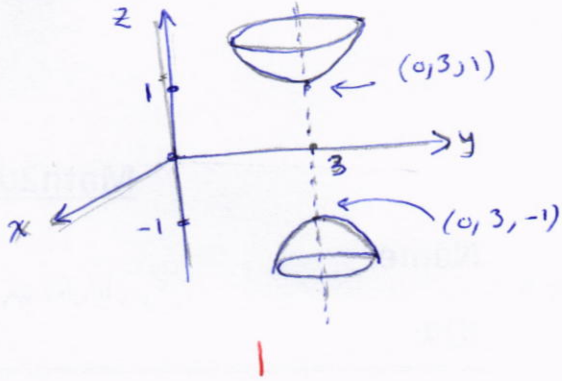
2) $z^2 - 19 = x^2 + 2y^2 - 12y$

$z^2 - 19 = x^2 + 2(y^2 - 6y + 9) - 18$

$z^2 - 19 = x^2 + 2(y-3)^2 - 18$

$-x^2 - 2(y-3)^2 + z^2 = 1$ 0.5

- a hyperboloid of two sheets. 0.5
- verteces: $(0,3,-1)$, $(0,3,1)$ 0.5
- axis: the line through $(0,3,\pm 1)$ and parallel to the z-axis. 0.5



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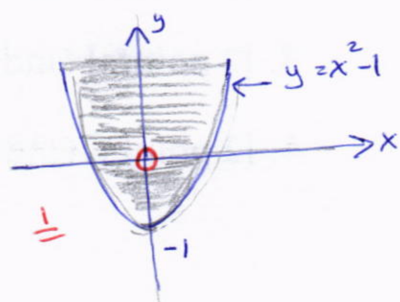
3) $f(x,y) = \dfrac{\sqrt{y - x^2 + 1}}{x^2 + y^2}$, $x^2+y^2=0 \Rightarrow (x,y)=(0,0)$

Domain $= \{(x,y) : y - x^2 + 1 \geq 0$ and $x^2+y^2 \neq 0\}$

$= \{(x,y) : y \geq x^2 - 1$ and $(x,y) \neq (0,0)\}$ 1

= all points (x,y) on and inside the parabola $y = x^2 - 1$, excluding the point $(0,0)$.



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4) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1 - \cos(3x^2+3y^2)}{\sin^2(x^2+y^2)}$, $\frac{0}{0}$, undefined : change to polar coordinates

$= \lim\limits_{r\to 0^+} \dfrac{1 - \cos(3r^2)}{\sin^2(r^2)}$ 0.5

$\stackrel{L'R}{=} \lim\limits_{r\to 0^+} \dfrac{\sin(3r^2)\cdot 6r}{2\sin(r^2)\cos(r^2)\cdot 2r}$ 0.5

$\stackrel{Simplif}{=} \lim\limits_{r\to 0^+} \dfrac{\sin(3r^2)}{\sin(r^2)} \cdot \dfrac{3}{2\cos(r^2)}$ $\longrightarrow \dfrac{3}{2(1)} = \dfrac{3}{2}$

L'R.

$= \lim\limits_{r\to 0^+} \dfrac{\cos(3r^2)\cdot 6r}{\cos(r^2)\cdot 2r} \cdot \dfrac{3}{2}$ 0.5

$= \lim\limits_{r\to 0^+} \dfrac{\cos(3r^2)}{\cos(r^2)} \cdot \dfrac{9}{2}$

$= \dfrac{1}{1} \cdot \dfrac{9}{2} = \dfrac{9}{2}$ 0.5