Math201.01, Quiz #1, Term 172

Name:

ID#: Solutions

Serial #:

---

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

$$x = 2 - sint, y = cost, 0 \leq t \leq 3\pi/2.$$

2. **[3 points]** Find the length of the polar curve $r = cos^2\left(\frac{\theta}{2}\right), 0 \leq \theta \leq \pi$.

3. **[4 points]** Let R be the region that lies inside both curves $r = \sqrt{3}cos\theta$ and $r = sin\theta$. Sketch the region R and find its area.

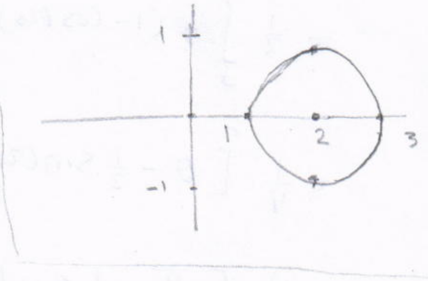
Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ As $\cos^2 t + \sin^2 t = 1$, then $y^2 + (2-x)^2 = 1$, or

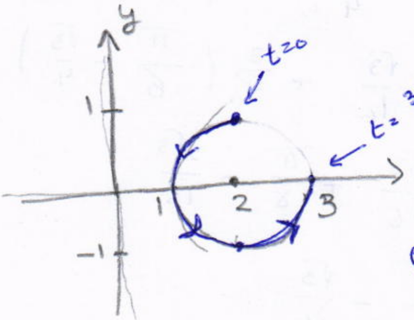
$(x-2)^2 + y^2 = 1$, a circle with center $(2, 0)$ and radius 1

1.5



Is it the full circle? Directions?

t	(x, y)
0	$(2, 1)$ ← initial point
$\frac{\pi}{2}$	$(1, 0)$
π	$(2, -1)$
$\frac{3\pi}{2}$	$(3, 0)$ ← terminal point



← The answer.

1.5

$\boxed{2}$ $$L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad 0.5$$

$$= \int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta$$

$$= 2 \cdot \sin\left(\frac{\theta}{2}\right)\Big]_0^{\pi}$$

$$= 2\left[1 - 0\right]$$

$$= 2 \quad 0.5$$

$$\bullet\ r = \cos^2\left(\frac{\theta}{2}\right) \Rightarrow \frac{dr}{d\theta} = 2\cos\left(\frac{\theta}{2}\right) \cdot -\sin\left(\frac{\theta}{2}\right) \cdot \frac{1}{2}$$

$$= -\cos\left(\frac{\theta}{2}\right) \cdot \sin\left(\frac{\theta}{2}\right) \quad 0.5$$

$$\bullet\ r^2 + \left(\frac{dr}{d\theta}\right)^2 = \cos^4\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)$$

$$= \cos^2\left(\frac{\theta}{2}\right)\left[\underbrace{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)}_{=1}\right]$$

$$= \cos^2\left(\frac{\theta}{2}\right) \quad 1$$

$$\bullet\ \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{\cos^2\left(\frac{\theta}{2}\right)} = \left|\cos\left(\frac{\theta}{2}\right)\right|$$

$$= \cos\left(\frac{\theta}{2}\right), \text{ as } 0 \le \theta \le \pi \Rightarrow 0 \le \frac{\theta}{2} \le \frac{\pi}{2}$$

$$\Rightarrow \frac{\theta}{2} \in 1^{st} \text{ quadrant}$$

0.5

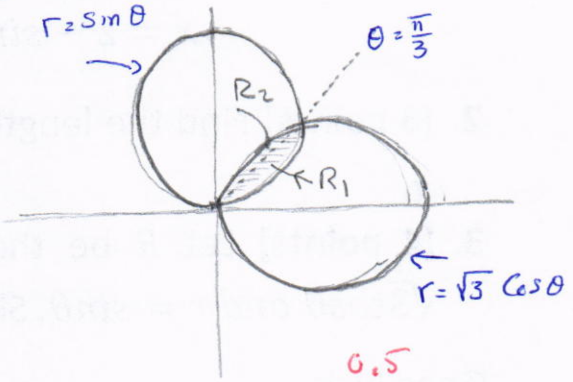
$\boxed{3}$. pts of intersection

$$\sin\theta = \sqrt{3}\cos\theta \Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = \frac{\pi}{3} \in 1^{st} \text{ quadrant}$$

(see the graph)

0.5



0.5

$$\text{Area} = R_1 + R_2$$

$$= \int_0^{\pi/3} \frac{1}{2}(\sin\theta)^2\, d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\sqrt{3}\cos\theta)^2\, d\theta \quad 1.5$$

$$= \frac{1}{2}\int_0^{\pi/3} \frac{1}{2}(1 - \cos(2\theta))\, d\theta + \frac{3}{2}\int_{\pi/3}^{\pi/2} \frac{1}{2}(1 + \cos(2\theta))\, d\theta \quad 1$$

$$= \frac{1}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\pi/3} + \frac{3}{4}\left[\theta + \frac{1}{2}\sin(2\theta)\right]_{\pi/3}^{\pi/2} \quad 0.5$$

$$= \frac{1}{4}\left[\frac{\pi}{3} - \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) - (0-0)\right] + \frac{3}{4}\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{1}{2}\sin\left(\frac{2\pi}{3}\right)\right)\right]$$

$$\boxed{\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{4}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) + \frac{3}{4}\left(\frac{\pi}{2} - \frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{16} + \frac{3}{4}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{16} + \frac{\pi}{8} - \frac{3\sqrt{3}}{16}$$

$$= \frac{5\pi}{24} - \frac{\sqrt{3}}{4}$$

Math201.02, Quiz #1, Term 172

Name:

ID#: **Serial #:**

---

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

$$x = \sqrt{t+1}, \qquad y = \sqrt{t-1}, \qquad t \geq 1.$$

2. **[3 points]** Find the length of the parametric curve

$$x = e^t + e^{-t}, y = 5 - 2t, 0 \leq t \leq 3.$$

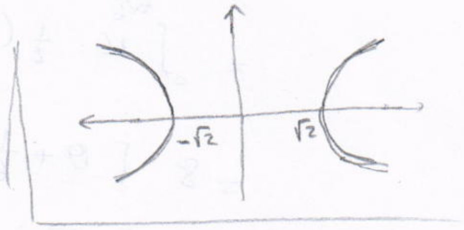
3. **[4 points]** Let R be the region enclosed by one loop of the curve $r = 4\cos(3\theta)$. Sketch the region R and find its area.

Good luck,

Ibrahim Al-Rasasi

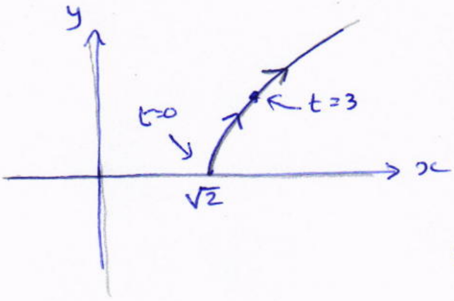
$\boxed{1}$ $x = \sqrt{t+1} \Rightarrow x^2 = t+1 \Rightarrow t = x^2 - 1$
$y = \sqrt{t-1} \Rightarrow y^2 = t-1 \Rightarrow t = y^2+1$ $\Big\} \Rightarrow x^2 - 1 = y^2 + 1$ 1.5

$\Rightarrow x^2 - y^2 = 2$, a horizontal hyperbola



. Is it the hyperbola? Direction?
Since $x = \sqrt{t+1}$ any $y = \sqrt{t-1}$ are positive,
then we take the part of the hyperbola
in the 1st quadrant:

t	(x, y)
1	$(\sqrt{2}, 0)$ ← initial point
3	$(2, \sqrt{2})$



$x \geq \sqrt{2}$
$y \geq 0$

← the answer

1.5

$\boxed{2}$ $L = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ 0.5

$= \int_0^3 e^t+e^{-t}\,dt$

$= e^t - e^{-t}\Big]_0^3$

$= (e^3-e^{-3})-(1-1)$

$= e^3-e^{-3}$ 0.5

• $x = e^t+e^{-t} \Rightarrow \frac{dx}{dt} = e^t - e^{-t}$ 0.5

$y = 5-2t \Rightarrow \frac{dy}{dt} = -2$

• $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 = (e^t-e^{-t})^2+(-2)^2$

$= e^{2t}-2+e^{-2t}+4$

$= e^{2t}+2+e^{-2t}$ 1

$= (e^t+e^{-t})^2$

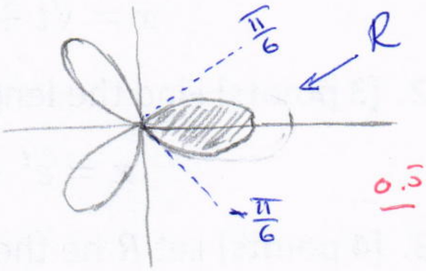
• $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = \sqrt{(e^t+e^{-t})^2} = e^t+e^{-t}$ 0.5

$\boxed{3}$ • $r=0 \Rightarrow 4\cos(3\theta)=0$

$\Rightarrow \cos(3\theta)=0$

$\Rightarrow 3\theta = \pm\frac{\pi}{2}$ (For the loop in Quadrant I, IV)

$\Rightarrow \theta = \pm\frac{\pi}{6}$ 0.5



0.5

$\text{Area} = \int_{-\pi/6}^{\pi/6} \frac{1}{2}\left(4\cos(3\theta)\right)^2\,d\theta$ 2

$\underline{\underline{\text{or}}} = 2\cdot\int_0^{\pi/6} \frac{1}{2}\left(4\cos(3\theta)\right)^2\,d\theta$, by Symmetry about the x-axis

$= \int_0^{\pi/6} 16\cos^2(3\theta)\,d\theta$

$= \int_0^{\pi/6} 16\cdot\frac{1}{2}\left(1+\cos(6\theta)\right)\,d\theta$ 0.5

$= 8\left[\theta+\frac{1}{6}\sin(6\theta)\right]_0^{\pi/6}$

$= 8\left[\left(\frac{\pi}{6}+0\right)-(0+0)\right]$

$= \frac{4\pi}{3}$ 0.5

⑤

Math201.04, Quiz #1, Term 172

Name: Solutions

ID#: **Serial #:**

---

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

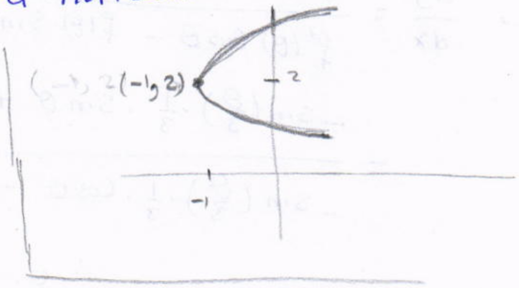
$$x = tan^2 t, \qquad y = 2 + sec t, \qquad 0 \le t < \pi/2.$$

2. **[3 points]** Find the area enclosed by the $y - axis$ and the parametric curve $x = 2t^2 - t, y = \sqrt{t}$.
3. **[2 points]** Sketch the set of all polar points (r, θ) such that $r \le -2$, $-\frac{\pi}{4} \le \theta < 0$.
4. **[2 points]** Find the slope of the tangent line to the polar curve $r = \cos(\frac{\theta}{3})$ at the point corresponding to $\theta = \pi$.

Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$. As $1+\tan^2 t = \sec^2 t$, then $1 + x = (y-2)^2$, a horizontal Parabola $\underline{\underline{1.5}}$

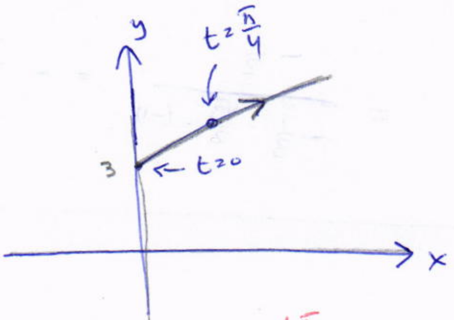


Is it the full parabola? Direction?

t	(x,y)
0	(0,3) ← initial point
$\frac{\pi}{4}$	$(1, 2+\sqrt{2})$

as $t \to \frac{\pi}{2}^-$, $(x,y) \to (\infty, \infty)$

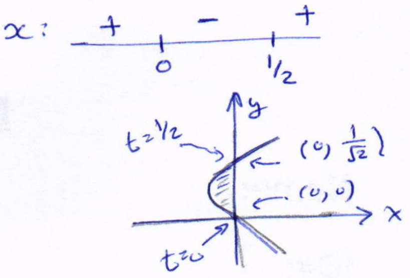
. Note that $x = \tan^2 t \ge 0$.



← the answer

$\underline{\underline{1.5}}$

$\boxed{2}$ Check the sign of x : $x = 2t^2 - t = t\,(2t-1) = 0 \implies t=0, \frac{1}{2}$



$$A = \int_0^{1/\sqrt{2}} -x\, dy$$

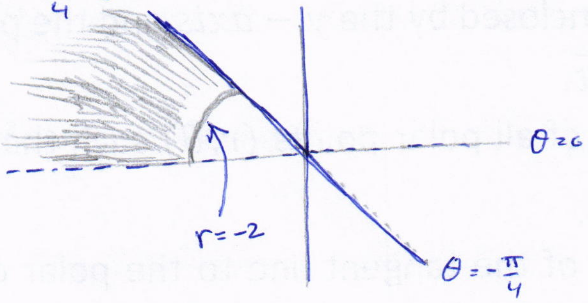
$$= \int_0^{1/2} -(2t^2-t)\cdot\frac{1}{2\sqrt{t}}\, dt$$

$$= -\frac{1}{2}\int_0^{1/2} (2t^{3/2} - t^{1/2})\, dt$$

$$= -\frac{1}{2}\left[2\cdot\frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2}\right]_0^{1/2} = -\frac{1}{2}\left(\frac{4}{5}\left(\frac{1}{2}\right)^{5/2} - \frac{2}{3}\left(\frac{1}{2}\right)^{3/2}\right]$$

$$= -\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{3/2}\left[\frac{4}{5}\cdot\frac{1}{2} - \frac{2}{3}\right] = -\frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\left(\frac{2}{5} - \frac{2}{3}\right) = -\left(\frac{1}{2}\right)^{3/2}\left(\frac{1}{5} - \frac{1}{3}\right) = -\left(\frac{1}{2}\right)^{3/2}\frac{-2}{15} = \frac{1}{15\sqrt{2}}$$

$\boxed{3}$ $r \le -2$, $-\frac{\pi}{4} \le \theta < 0$



$\boxed{4}$ $r = \cos\left(\frac{\theta}{3}\right)$, $\theta = \pi$. $f(\theta) = \cos\left(\frac{\theta}{3}\right)$

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} \qquad 0.5$$

$$= \frac{-\sin\left(\frac{\theta}{3}\right)\cdot\frac{1}{3}\cdot\sin\theta + \cos\left(\frac{\theta}{3}\right)\cdot\cos\theta}{-\sin\left(\frac{\theta}{3}\right)\cdot\frac{1}{3}\cdot\cos\theta - \cos\left(\frac{\theta}{3}\right)\sin\theta} \qquad 0.5$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{0 + \cos\left(\frac{\pi}{3}\right)\cos(\pi)}{-\frac{1}{3}\sin\left(\frac{\pi}{3}\right)\cos\pi} \qquad 0.5$$

$$= \frac{-\frac{1}{2}\ \cancel{3}}{-\frac{1}{3}\cdot\frac{\sqrt{3}}{2}\cdot(-1)} = -\frac{3}{\sqrt{3}} = -\sqrt{3} \qquad 0.5$$