KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Department of Mathematics and Statistics

Math 201 Second Major Exam

Semester (172)

April 04, 2018 at 06:15 PM-08:15 PM

KEY

Name: ....

I.D: .... **Section:** .... **Serial #:** ....

Instructions

1. No electronic device (such as calculator, mobile phone, smart watch) is allowed in this exam.
2. Justify your answers. No credit is given for (correct) answers not supported by work.

Question	Points
1	/15
2	/12
3	/14
4	/15
5	/11
6	/16
7	/17
Total	/100

Question 1 (4+5+6 points)

Consider the following two lines:

$$L_1: x = 2t+1, \qquad y = 3t+2, \qquad z = 4t+3$$

$$L_2: x = s+2, \qquad y = 2s+4, \qquad z = -4s-1.$$

a) Find the point at which the line L_2 intersects the $xz-plane$.

b) Find the point of intersection of the lines L_1 and L_2.

c) Find the equation of the plane that contains the lines L_1 and L_2.

a) on the yz-plane $y=0$. 1

$2s+4=0 \Rightarrow s=-2$ 1

$x=-2+2=0$, $z=-4(-2)-1=7$

the point of intersection is $(0,0,7)$ 2

b) We solve the system $\begin{cases} 2t+1=s+2 \\ 3t+2=2s+4 \\ 4t+3=-4s-1 \end{cases} \Rightarrow$ 1

We find $t=0$ and $s=-1$. 2

Thus $x=1$, $y=2$, $z=3$, and the point of intersection is $(1,2,3)$. 2

c) $\vec{V_1}=\langle 2,3,4\rangle$ $\vec{V_2}=\langle 1,2,-4\rangle$ 2

the normal vector is $\vec{n}=\vec{V_1}\times\vec{V_2}=\langle -20,12,1\rangle$ 2

a point on the plane is $(1,2,3)$ 1

an equation of the plane is

$$20x-12y-z+7=0$$ 2

Question 2 (5+7 points)

a) Describe all traces of the surface $x - y^2 - 4z^2 = 0$, that are parallel to the $yz - plane$.

The planes $x = k$, $k \in \mathbb{R}$ are parallel to the yz-plane.

The traces on $x=k$ is $y^2+4z^2=k$ 2

if $k<0$, then there is no trace. 1

if $k=0$, then the point $(0,0,0)$ is the trace 1

if $k>0$, then the trace is the ellipse $y^2+4z^2=k$. 1

b) Identify (name, axis, vertex) and sketch the surface

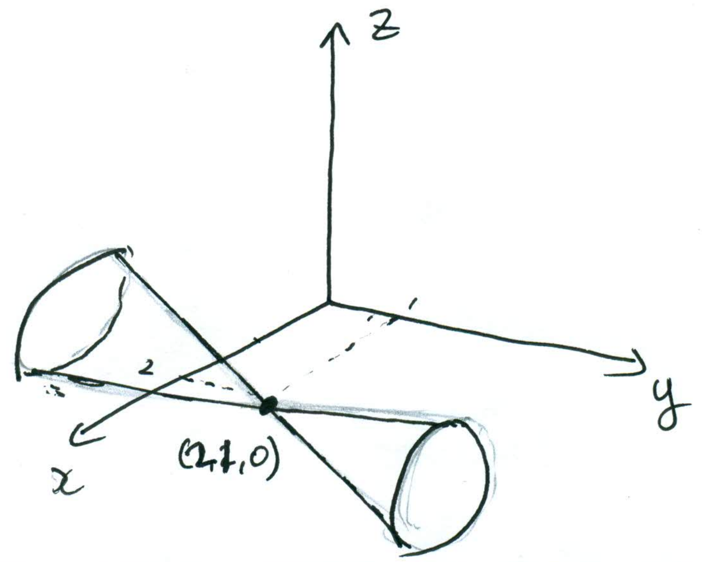
$$-x^2 + 4x + y^2 - 2y - z^2 - 3 = 0.$$

We can rewrite the surface eqn as $(x-2)^2+z^2=(y-1)^2$ 2

name: cone 1

axis: parallel to the y-axis. 1

Vertex: $(2,1,0)$ 1



2

Question 3 (8+6 points)

Consider the function $f(x,y) = \sqrt{x^2 + \frac{1}{4}y^2 - 2x + y + 1}\,.$

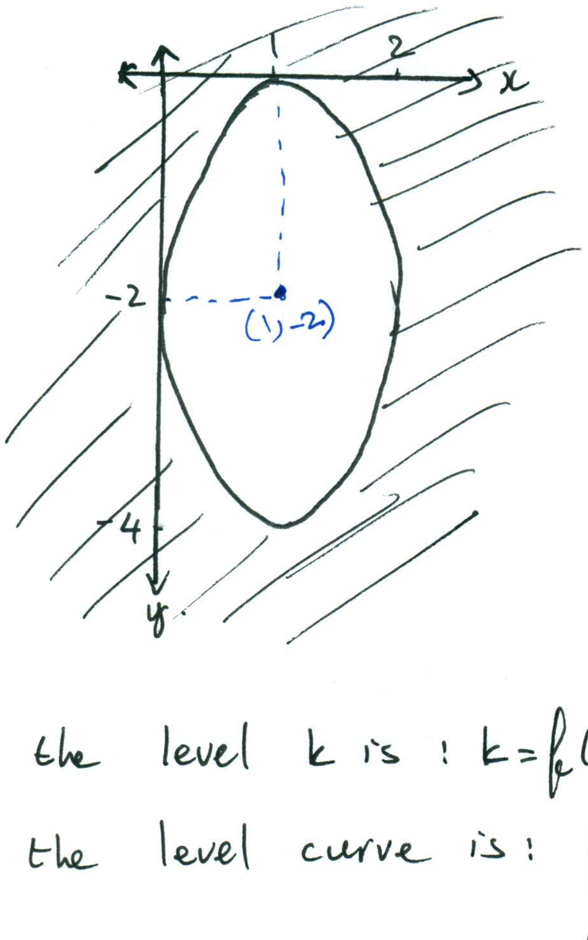
a) Find and sketch the domain of f.

b) Find the $x - intercept(s)$ of the level curve of f that passes through the point $P(3,-2)$.

a) The domain is

$$D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + \frac{1}{4}y^2 - 2x + y + 1 \geq 0 \} \quad 2$$

$$= \{ (x,y) \in \mathbb{R}^2 \mid (x-1)^2 + \frac{1}{4}(y+2)^2 - 1 \geq 0 \} \quad 2$$



ellipse — 2

region — 2

b) the level k is : $k = f(3,-2) = \sqrt{9+1-6-2+1} = \sqrt{3}$ 2

the level curve is : $f(x,y) = \sqrt{3}$

$$(x-1)^2 + \frac{1}{4}(y+2)^2 = 4 \quad 2$$

the x-intercept is: $(x-1)^2 + \frac{1}{4}(0+2)^2 = 4$

$$x = 1 \pm \sqrt{3} \quad 2$$

Question 4 (7+8 points)

a) Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}.$$

along the line $x=1$, $\lim_{y\to 0} \frac{0}{y^2} = 0$ $1+2=3$

along the line $y=x-1$, $\lim_{y\to 0} \frac{y^2}{2y^2} = \frac{1}{2}$ $1+2=3$

thus the limit DNE. 1

b) If the function $f(x,y) = \begin{cases} \frac{x^2 \sin^2 y}{x^2+2y^2}, & (x,y) \neq (0,0) \\ k, & (x,y) = (0,0) \end{cases}$ is continuous

at (0,0), find k.

If f is continuous at $(0,0)$, then $\lim_{(x,y)\to(0,0)} f(x,y) = k$ 2

$0 \leqslant \frac{x^2}{x^2+2y^2} \leqslant 1.$ 2

$0 \leqslant \frac{x^2 \sin^2 y}{x^2+2y^2} \leqslant \sin^2 y$ 1

$\lim_{y\to 0} \sin^2 y = 0.$ 1

Then by the Squeeze Thm. $\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2+2y^2} = 0.$ 1

So $k = 0.$ 1

Question 5 (5+6 points)

a) Let $f(x,y) = (xy)^x$. Find $f_{xy}(1,2)$.

$f_x(x,y) = (xy)^x (\ln x + \ln y + 1)$ 2

$f_{xy}(x,y) = \frac{(xy)^x}{y} + x^2(xy)^{x-1}(\ln x + \ln y + 1)$ 2

$f_{xy}(1,2) = \frac{2^1}{2} + 1 \cdot 2^0 (\ln 1 + \ln 2 + 1) = 2 + \ln 2$ 1

b) Suppose $f(x,y)$ has continuous first partial derivatives and the equation of the tangent plane at the point $P(1,1,1)$ on its surface is given by $2x + 3y + 5z = 10$. Use linearization at P to approximate $f(1.1,1.1)$.

$L(x,y) = \frac{1}{5}(10 - 2x - 3y)$ 3

$f(1.1, 1.1) \approx L(1.1, 1.1) = \frac{1}{5}(10 - 2.2 - 3.3)$ 3

$= 0.9$

Question 6 (8+8 points)

a) If $u = \sqrt[3]{r^3 + s^3}$, $r = y + x\cos t$, $s = x + y\sin t$.

Find $\frac{\partial u}{\partial y}$ when $x = 1$, $y = 2$ and $t = 0$.

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y}$$ 2

$$= \frac{1}{3}(r^3+s^3)^{-2/3}.3r^2 + \frac{1}{3}(r^3+s^3)^{-2/3}.3s^2.\sin t$$

$$= (r^3+s^3)^{-2/3}(r^2 + s^2\sin t)$$ 2

$x=1,\ y=2,\ t=0 \Rightarrow r=3,\ s=1$ 2

$$\left.\frac{\partial u}{\partial y}\right|_{x=1,y=2,t=0} = 28^{-2/3}(9)$$ 2

b) Suppose that z is a function of x and y given implicitly by the equation $e^z = xyz$. Show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$.

$$F(x,y,z) = e^z - xyz.$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{-yz}{e^z - xy} = \frac{yz}{e^z - xy}$$ 3

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-xz}{e^z - xy} = \frac{xz}{e^z - xy}$$ 3

Then $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \frac{xyz}{e^z - xy} - \frac{xyz}{e^z - xy} = 0$ 2

Question 7 (9+8 points)

a) Find the directional derivative of $f(x,y) = \sqrt{xy}$ at the point $P(2,8)$ in the direction of the point $Q(5,4)$.

$\overrightarrow{PQ} = \langle 3, -4 \rangle$. 2

$\nabla f(x,y) = \langle \frac{1}{2}\sqrt{\frac{y}{x}}, \frac{1}{2}\sqrt{\frac{x}{y}} \rangle$ 2

$\nabla f(2,8) = \langle 1, \frac{1}{4} \rangle$ 1

unit vector in the required direction $\vec{u} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{1}{5}\langle 3,-4 \rangle$ 1

directional derivative:

$D_{\vec{u}} f(2,8) = \frac{1}{5}\langle 3,-4 \rangle \cdot \langle 1, \frac{1}{4} \rangle = \frac{2}{5}$ 2

b) Find the point of intersection between the normal line to the ellipsoid $4x^2 + y^2 + 4z^2 = 12$ at the point $P(1,2,1)$ and the plane $x + y + z = 8$.

$f(x,y,z) = 4x^2 + y^2 + 4z^2 - 12$

$\nabla f(x,y,z) = \langle 8x, 2y, 8z \rangle$, $\nabla f(1,2,1) = \langle 8,4,8 \rangle$ 2

parametric equations of the normal line $\begin{cases} x = 1+8t \\ y = 2+4t \\ z = 1+8t \end{cases}$ 2

finding the intersection:

$1+8t + 2+4t + 1+8t = 8 \Rightarrow 20t = 4 \Rightarrow t = \frac{1}{5}$ 2

$x = \frac{13}{5}$, $y = \frac{14}{5}$, $z = \frac{13}{5}$

the point of intersection is $\left(\frac{13}{5}, \frac{14}{5}, \frac{13}{5}\right)$. 2