DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

MATH 131: FINITE MTHEMATICS

KEY

Semester 172
Major Exam One
Tuesday, February 20, 2018
Allowed time 75 minutes

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Section 1

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Section 2

Name:

ID#:

Serial #:

Section:

Directions:

1) You must **show all your work** to obtain full credit.
2) You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam.
3) Do not keep your mobile with you during the exam, turn off your mobile and leave it aside.

Question No	Full Marks	Marks Obtained
Q1	6	
Q2	5	
Q3	4	
Q4	6	
Q5	5	
Q6	5	
Q7	5	
Q8	4	
Total	40	

1. A firm manufactures a commodity that costs $20 per unit to produce. In addition, the firm has fixed costs of $2,000. Each unit is sold for $70.

a. How many units must be sold if the firm is to meet a profit target of $14,500?

Let q be the No. of units

$\text{profit} = \text{Total Revenue} - \text{Total Cost}$ } (1) (2 marks)

$14{,}500 = 70q - 20q - 2{,}000$

$50q = 16500$ } (1)

$q = 330$

b. Find the break-even quantity.

(2 marks)

$\text{profit} = 0$

$\Rightarrow TC = TR$ } (1)

$20q + 2000 = 70q$

$50q = 2000$ } (1)

$q = 40$

c. How many units must be sold if the firm wants to earn a profit of more than $50,000?

(2 marks)

$\text{profit} > 50{,}000$

$70q - 20q - 2000 > 50{,}000$ } (1)

$50q > 52{,}000$

$q > 1040$ } (1)

OR $q \geq 1041$

2. A manufacturer produces 50 TV sets at a cost of \$17,500 and 75 TV sets at a cost of \$21,250.

a. To describe this situation, write an equation of a line in slope-intercept form $(C = mq + b)$

$(50, 17,500)$ and $(75, 21250)$.

$$m = \frac{21250 - 17,500}{75 - 50} = 150$$ (1)

3 marks

$$C - 17,500 = 150(q - 50) \Rightarrow C = 150q + 10,000$$ (1) (1)

b. How much average cost will increase per TV set?

\$ 150 (1)

1 marks

c. Predict the cost to produce 120 TV sets.

$$C = 150(60) + 10,000 = 19,000$$

1 marks

3. A revenue function is given by $R(q) = 15 + 114q - 3q^2$, where q is the number of units produced and sold.

a. Find the value of q for which the revenue is maximum.

$$q = -\frac{b}{2a} = \frac{-114}{2(-3)} = 19$$

2 marks

b. What is the maximum revenue?

2 marks

$$R = f\left(-\frac{b}{2a}\right) = f(19) = 15 + 114(19) - 3(19)^2 = 1,098$$

3 marks

4. Suppose that the supply and demand equations for a certain product are

$$p = \frac{1}{14}q - 9 \text{ and } p = -\frac{1}{70}q + 3 \text{ Respectively.}$$

Where p represents the price per unit in dollars and q represents the number of units per time period.

a. Find the equilibrium price.

S. Eq $\Rightarrow$ $p = \frac{1}{14}q - 9$ ⟶ (1)

D. Eq $\Rightarrow$ $p = -\frac{1}{70}q + 3$ ⟶ (2)

put (1) in (2)

$$\frac{1}{14}q - 9 = -\frac{1}{70}q + 3 \quad (1)$$

$$\left(\frac{1}{14} + \frac{1}{70}\right)q = 12$$

$$q = 12 \times \frac{980}{84} = 140 \quad (1) \text{ put in } (1)$$

(1) $\Rightarrow$ $p = \frac{1}{14}(140) - 9 = \$1$ (1)

3 marks

b. Find the equilibrium price when a tax of \$3 per unit is imposed on the manufacturer if the demand remains the same.

After Tax S. Eq $\Rightarrow$ $p = \frac{1}{14}q - 9 + 3$ ⟶ (1) put in (2)

(2) $\Rightarrow$ $\frac{1}{14}q - 6 = -\frac{1}{70}q + 3$ (1)

$$\frac{84}{980}q = 9$$

$$q = 105 \quad (1) \text{ put in } (1)$$

(1) $\Rightarrow$ $p = \frac{1}{14}(105) - 6 = \$1.5$ (1)

5. Solve the system $\begin{cases} 2x-5y=10 \\ y=\sqrt{x+4} \end{cases}$ $\Rightarrow$ $y = \frac{2}{5}x - 2$ → (1) put in (2)

→ (2)

$\left(\frac{2}{5}x-2\right)^2 = \left(\sqrt{x+4}\right)^2$ (1)

$\frac{4}{25}x^2 - \frac{8}{5}x + 4 = x + 4$ (1)

$\frac{4}{25}x^2 - \frac{8}{5}x - x = 0$

$4x^2 - 40x - 25x = 0$ (1)

$4x^2 - 65x = 0$

$x(4x-65) = 0$

$x = 0$ or $x = 65/4 = 16.25$ put in (1)

(1) $\Rightarrow$ $y = -2$ (1) $\Rightarrow$ $y = 9/2 = 4.5$ (1)

$(0, -2)$ is one solution $(65/4, 9/2)$ $(16.25, 4.5)$ is the other solution

5 marks

6. A manufacturer produces two products, A and B. For each unit of A sold the profit is \$8. For each unit of B sold the profit is \$11. From past experience it has been found that 25 percent more of A can be sold than of B. Next year the manufacturer desires a total profit of \$42,000. How many units of each product must be sold?

5 marks

Let x and y be the No. of units of product A and B sold respectively. (1)

$8x + 11y = 42{,}000$ → (1)

$x = y + 0.25y = 1.25y$ → (2) put in (1) (1)

(1) $\Rightarrow$ $8(1.25y) + 11y = 42{,}000$ (1)

$21y = 42{,}000 \Rightarrow y = 2{,}000$ (1)

5 marks

7. For what values of a will the following system of equations have a solution?

$$x - y - 3z = 2 \quad (1)$$
$$x + y - z = 1 \quad (2)$$
$$2x - y - 5z = a \quad (3)$$

Adding (1) and (2)

$$x - y - 3z = 2$$
$$x + y - z = 1$$
$$2x - 4z = 3 \quad (4)$$ (1.5)

Adding (2) & (3)

$$x + y - z = 1$$
$$2x - y - 5z = a$$
$$3x - 6z = a + 1 \quad (5)$$ (1.5)

(4) $\times -3 \Rightarrow -6x + 12z = -9$

(5) $\times 2 \Rightarrow 6x - 12z = 2(a+1)$ (1) (2)

$$0 = 2a + 2 - 9 \Rightarrow a = 7/2 = 3.5$$ (1)

8. Write the matrix $\begin{bmatrix} 2 & 2 & 4 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ in reduced form

4 marks

(1) R_{13} $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ $-R_1 + R_2$, $-2R_1 + R_3$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$ (2)

$-2R_2 + R_3 \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (1)