KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **6B**

Name: Solution ID #: ______ Section #: ______

Q1: (4 *points*) Determine whether the series is convergent or divergent. (Use Ratio Test or Root Test)

$$(i) \sum_{k=1}^{\infty} \frac{n^2(n+2)!}{n!\,3^{2n}}$$

let $a_n = \dfrac{n^2(n+2)!}{n!\,3^{2n}}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty} \frac{(n+1)^2(n+3)!}{(n+1)!\,3^{2(n+1)}} \cdot \frac{n!\,3^{2n}}{n^2(n+2)!}$$

$$= \lim_{n\to\infty} \frac{(n+1)(n+3)}{n^2\,3^2} = \frac{1}{9} < 1$$

Thus, by the Ratio Test, the series is absolutely convergent and therefore convergent

$$(ii) \sum_{n=1}^{\infty} \frac{6}{\left(2+\left(\frac{1}{n}\right)\right)^{2n}}$$

$$= 6\sum_{n=1}^{\infty}\left[\frac{1}{\left(2+\frac{1}{n}\right)^2}\right]^n$$

Let $a_n = \left[\dfrac{1}{\left(2+\frac{1}{n}\right)^2}\right]^n$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1}{\left(2+\frac{1}{n}\right)^2}$$

$$= \frac{1}{4} < 1$$

So, $\sum_{n=1}^{\infty} a_n$ is Convergent by the Root Test.

Hence, $\sum_{n=1}^{\infty} \dfrac{6}{\left(2+\frac{1}{n}\right)^{2n}}$ is Convergent

Q2: (2 *points*) Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$$

put $b_n = \sqrt{n+1} - \sqrt{n}$

$$= \left(\sqrt{n+1}-\sqrt{n}\right)\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

(i) $b_{n+1} = \dfrac{1}{\sqrt{n+2}+\sqrt{n+1}} < b_n$,

for $n \geq 1$,

b_n is decreasing.

(ii) $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n+1}+\sqrt{n}}$

$$= 0$$

So, $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$

is Convergent by the Alternating Series Test.

Q3: (4 *points*) Determine whether the series converges or diverges. (Use Comparison or Limit Comparison Test)

(*i*) $\displaystyle\sum_{n=1}^{\infty} \frac{8n+1}{n(n+1)(n+2)}$

let $a_n = \dfrac{8n+1}{n(n+1)(n+2)}$

choose $b_n = \dfrac{8n}{n^3} = \dfrac{8}{n^2}$

Σb_n is convergent

(a constant multiple of a convergent p-series $p=2$)

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{8n^3+n^2}{8n^3+24n^2+16n}$$

$= 1 > 0$

Thus,

$\displaystyle\sum_{n=1}^{\infty} \frac{8n+1}{n(n+1)(n+2)}$

is also convergent by The Limit Comparison Test

(*ii*) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{\sqrt{n}\ln n}$

Since $\sqrt{n} > \ln n$ for $n \geq 2$,

$$\frac{1}{\sqrt{n}\ln n} > \frac{1}{\sqrt{n}\sqrt{n}} = \frac{1}{n}$$

let $a_n = \dfrac{2}{\sqrt{n}\ln n}$, $b_n = \dfrac{1}{n}$

$a_n > b_n$ for $n \geq 2$.

Since $\displaystyle\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{1}{n}$

is divergent (Harmonic Series),

$\displaystyle\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$

is also divergent by the Comparison Test.