KFUPM – Department of Mathematics & Statistics
MATH102 – Term 172
Quiz 5B (Duration: 25 Minutes)

Name: ______________________ ID #: ______________________ Section #: ______

Q1:

a) (*1 point*) Determine whether the sequence is increasing, decreasing or not monotone. Is the sequence bounded? (Justify your answer)

$$a_n = 1 + \frac{(-1)^n}{n}$$

If we write out the terms of the Sequence, we obtain:

$$\left\{0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \ldots\right\}.$$

The Sequence is not Monotonic.

Since $0 \le a_n \le \frac{3}{2}$ for all n,

the sequence is bounded.

b) (*2 points*) Determine whether the sequence converges or diverges. If it converges, find the limit.

(*i*) $a_n = \cos\left(\frac{n\pi}{n+1}\right)$

We know that

$$\lim_{n\to\infty} \frac{n\pi}{n+1} = \pi$$

and the cosine function is continuous at π.

Thus,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\left(\frac{n\pi}{n+1}\right)$$

$$= \cos\left(\lim_{n\to\infty} \frac{n\pi}{n+1}\right)$$

$$= \cos \pi$$

$$= -1$$

(*ii*) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$

If we write out the terms of the Sequence, we get

$$\{1, 0, -1, 0, 1, 0, -1, \ldots\}.$$

Since the terms oscillate infinitely often, it does not approach any number.

Thus, $\lim_{n\to\infty} \sin \frac{n\pi}{2}$ does not exist.

The Sequence is divergent

Q2: (3.5 *points*) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(i) $\sum_{n=1}^{\infty} \frac{1+4n^2}{3n^2+2n}$

Since

$$\lim_{n\to\infty} \frac{1+4n^2}{3n^2+2n} = \frac{4}{3} \neq 0,$$

the given series is divergent by the Test for Divergence

(ii) $\sum_{k=1}^{\infty} (\sqrt{2})^{-k}$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^{k}$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)^{k-1}.$$

It is a geometric series with

$a = \frac{1}{\sqrt{2}}$, $r = \frac{1}{\sqrt{2}}$.

The Series is Convergent: $|r| < 1$,

and $\sum_{k=1}^{\infty} (\sqrt{2})^{-k} = \frac{1/\sqrt{2}}{1 - 1/\sqrt{2}} = \frac{1}{\sqrt{2}-1}$

Q3: (3.5 *points*) Use the Integral Test to determine whether the series is convergent or divergent.

(i) $\sum_{k=1}^{\infty} ke^{-k^2}$

Let $f(x) = xe^{-x^2}$.

f is continuous, positive and decreasing ($\frac{df}{dx} < 0$ for $x \geq 1$).

$$\lim_{t\to\infty} \int_1^t xe^{-x^2}dx = \left[-\frac{e^{-x^2}}{2}\right]_1^t$$

$$= \lim_{t\to\infty} \left[-\frac{e^{-t^2}}{2} + \frac{1}{2}e^{-1}\right]$$

$$= \frac{1}{2e}$$

So, the Series is Convergent.

(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}}$

Let $f(x) = \frac{1}{\sqrt{x+3}}$ → positive, continuos and decreasing on $[1, \infty)$

$$\int_1^{\infty} \frac{1}{\sqrt{x+3}}dx = \lim_{t\to\infty} \left[2\sqrt{x+3}\right]_1^t$$

$$= \lim_{t\to\infty} \left[2\sqrt{t+3} - 4\right]$$

→ divergent

So the Series is divergent.