KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **4C** (Duration: 25 Minutes)

Name: Solution ID #: ______ Section #: ______

Q1: (3 *points*) Evaluate

Put $u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$\theta = 0 \iff u = 1$

$\theta = \pi/3 \iff u = 2$

$$I = \int_0^{\pi/3} \tan^5\theta \sec^6\theta\, d\theta = \int_0^{\pi/3} \tan^4\theta \sec^5\theta \sec\theta\tan\theta\, d\theta$$

$$= \int_0^{\pi/3} (\sec^2\theta - 1)^2 \sec^5\theta \sec\theta \tan\theta\, d\theta$$

$$I = \int_1^2 (u^2-1)^2 u^5\, du = \int_1^2 (u^9 - 2u^7 + u^5)\, du$$

$$= \left[\frac{u^{10}}{10} - \frac{u^7}{4} + \frac{u^6}{6}\right]_1^2 = \frac{981}{20}$$

Q2: (3.5 *points*) Evaluate

$$I = \int \sqrt{2 + 2x - x^2}\, dx = \int \sqrt{3 - (x-1)^2}\, dx$$

Put $x - 1 = \sqrt{3}\sin\theta$

$dx = \cos\theta\, d\theta$

$$= \sqrt{3}\int \sqrt{3 - 3\sin^2\theta}\, \cos\theta\, d\theta$$

$$I = 3\int \cos^2\theta\, d\theta = \frac{3}{2}\int (1 + \cos 2\theta)\, d\theta = \frac{3}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

$$= \frac{3}{2}\left[\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) - \frac{1}{2}\,\frac{(x-1)\sqrt{2+2x-x^2}}{3}\right] + C$$

Q3: (3.5 *points*) Evaluate the integral

$$I = \int \frac{x^2 - 2x - 1}{(x-1)(x^2+2)}\, dx = \int \left(\frac{A}{x-1} + \frac{Bx + C}{x^2+2}\right) dx$$

$Ax^2 + 2A + Bx^2 - Bx + Cx - C = x^2 - 2x - 1$

$$\left.\begin{aligned} A + B &= 1 \\ 2A - C &= -1 \\ C - B &= -2 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= -2/3 \\ B &= 5/3 \\ C &= -1/3 \end{aligned}$$

$$I = \int \left(\frac{-2/3}{x-1} + \frac{5/3\, x}{x^2+2} + \frac{-1/3}{x^2+2}\right) dx$$

$$= -\frac{2}{3}\ln|x-1| + \frac{5}{6}\ln(x^2+2) - \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **4B** (Duration: 25 Minutes)

Name: Solution ID #: ____________ Section #: _____

Q1: (3 *points*) Evaluate

$$\boxed{\begin{array}{l}\text{Put } u = \sin\theta \\ du = \cos\theta \, d\theta\end{array}}$$

$$I = \int \cot^5\theta \sin^4\theta \, d\theta = \int \frac{\cos^5\theta}{\sin\theta} \, d\theta$$

$$= \int \frac{(1-\sin^2\theta)^2 \cos\theta}{\sin\theta} \, d\theta$$

$$I = \int \frac{(1-u^2)^2}{u} \, du = \int \left(\frac{1}{u} - 2u + u^3\right) du = \ln|u| - u^2 + \frac{u^4}{4} + C$$

$$= \ln|\sin\theta| - \sin^2\theta + \frac{\sin^4\theta}{4} + C$$

Q2: (3.5 *points*) Evaluate

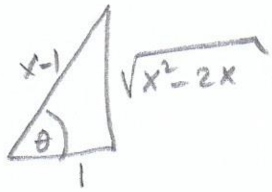
$$I = \int \sqrt{x^2 - 2x} \, dx = \int \sqrt{(x-1)^2 - 1} \, dx \qquad \boxed{\begin{array}{l}\text{Put } u = x-1 \\ du = dx\end{array}}$$

$$I = \int \sqrt{u^2 - 1} \, du. \qquad \boxed{\begin{array}{l}\text{Put } u = \sec\theta \\ du = \sec\theta \tan\theta \, d\theta\end{array}}$$

$$= \int \sqrt{\sec^2\theta - 1} \, \sec\theta \tan\theta \, d\theta$$

$$= \int \sec\theta \tan^2\theta \, d\theta = \frac{1}{2}\left(\sec\theta \tan\theta - \ln|\sec\theta + \tan\theta|\right) + C$$

$$= \frac{1}{2}\left[(x-1)\sqrt{x^2-2x} - \ln\left|(x-1) + \sqrt{x^2-2x}\right|\right] + C$$



Q3: (3.5 *points*) Evaluate the integral

$$I = \int \frac{x^2 - 2x - 1}{(x-1)(x^2+1)} \, dx = \int \left(\frac{A}{x-1} + \frac{Bx + C}{x^2+1}\right) dx$$

$$Ax^2 + A + Bx^2 - Bx + Cx - C = x^2 - 2x - 1$$

$$\left.\begin{array}{l} A + B = 1 \\ -B + C = -2 \\ A - C = -1 \end{array}\right\} \Rightarrow \begin{array}{l} A = -1 \\ B = 2 \\ C = 0 \end{array}$$

$$I = \int \left(-\frac{1}{x-1} + \frac{2x}{x^2+1}\right) dx$$

$$= -\ln|x-1| + \ln(x^2+1) + C$$

$$= \ln\left|\frac{x^2+1}{x-1}\right| + C$$

KFUPM – Department of Mathematics & Statistics
MATH102 – Term 172
Quiz **4A** (Duration: 25 Minutes)

Name: Solution ID #: ______________ Section #: _____

Q1: (3 *points*) Evaluate

$$I = \int_0^{\pi/4} \frac{\sin\theta}{\cos^5\theta}\,d\theta.$$

Put $u = \cos\theta$,
$du = -\sin\theta\, d\theta$
$\theta = 0 \Rightarrow u = 1$
$\theta = \pi/4 \Rightarrow u = 1/\sqrt{2}$

$$I = -\int_1^{1/\sqrt{2}} \frac{du}{u^5} = \left[\frac{1}{4u^4}\right]_1^{1/\sqrt{2}}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Q2: (3.5 *points*) Evaluate

$$I = \int \sqrt{x^2+4x}\,dx = \int \sqrt{(x+2)^2-4}\,dx.$$

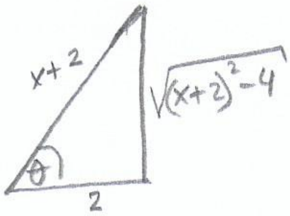
Put $u = x+2$
$du = dx.$

$$I = \int \sqrt{u^2-4}\,du,$$

Put $u = 2\sec\theta$
$du = 2\sec\theta\tan\theta\,d\theta$

$$= \int \sqrt{4\sec^2\theta - 4}\,(2\sec\theta\tan\theta\,d\theta) = 4\int \sec\theta\tan^2\theta\,d\theta$$

$$= 2\left(\sec\theta\tan\theta - \ln|\sec\theta + \tan\theta|\right) + C$$



$$= 2\left[\frac{(x+2)\sqrt{x^2+4x}}{4} - \ln\left|\frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2}\right|\right] + C$$

$$= \frac{(x+2)\sqrt{x^2+4x}}{2} - 2\ln\left|\frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2}\right| + C$$

Q3: (3.5 *points*) Evaluate the integral

$$I \int \frac{x^2-2x}{(x+1)(x^2+1)}dx = \int\left[\frac{A}{x+1} + \frac{Bx+C}{x^2+1}\right]dx$$

$Ax^2 + A + Bx^2 + Bx + Cx + C = x^2 - 2x$

$A + C = 0 \Rightarrow A = -C$
$A + B = 1$
$B + C = -2$

$B = -1/2$
$C = -3/2$
$A = 3/2$

$$I = \int\left(\frac{3/2}{x+1} - \frac{\frac{1}{2}x}{x^2+1} - \frac{3/2}{x^2+1}\right)dx$$

$$= \frac{3}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) - \frac{3}{2}\tan^{-1}(x) + K$$

K is an arbitrary constant.