KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **3A** (Duration: 25 Minutes)

Name: Solution ID #: ______ Section #: ______

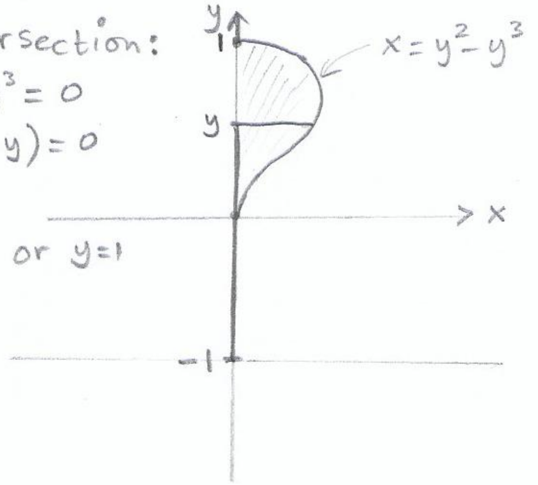
Q1: (4 *points*) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve $x = y^2 - y^3$ and the line $x = 0$ about the line $y = -1$.

Intersection:

$y^2 - y^3 = 0$

$y^2(1-y) = 0$

$y = 0$ or $y = 1$



Circumference : $2\pi(y+1)$

Height : $y^2 - y^3$

$$V = \int_0^1 2\pi(y+1)(y^2-y^3)\,dy$$

$$= 2\pi \int_0^1 (y^3 + y^2 - y^4 - y^3)\,dy$$

$$= 2\pi \left(\tfrac{1}{3}y^3 - \tfrac{1}{5}y^5\right)\Big|_0^1 = 2\pi\left(\tfrac{1}{3} - \tfrac{1}{5}\right)$$

$$= \frac{4\pi}{15}$$

Q2: (2 *points*) Find the average value of $f(x) = 2x + \sqrt{x}$ on the interval $[1, 4]$.

$$f_{ave} = \frac{1}{4-1}\int_1^4 (2x + \sqrt{x})\,dx$$

$$= \frac{1}{3}\left(x^2 + \frac{2}{3}x^{3/2}\right)\Big|_1^4$$

$$= \frac{1}{3}\left[16 + \frac{2}{3}\left(4^{3/2}\right) - \left(1 + \frac{2}{3}\right)\right]$$

$$= \frac{1}{3}\left[16 + \frac{2}{3}(8) - \frac{5}{3}\right]$$

$$= \frac{1}{3}\left[16 + \frac{16}{3} - \frac{5}{3}\right]$$

$$= \frac{1}{3}\left[16 + \frac{11}{3}\right]$$

$$= \frac{59}{9}$$

Q3: (4 *points*) Evaluate the integral

(a) $\int t\csc^2(2t)\,dt = I$

$u = t$, $du = dt$

$dv = \csc^2(2t)\,dt$

$v = -\frac{1}{2}\cot(2t)$

$$I = -\frac{t}{2}\cot(2t) + \int \frac{1}{2}\cot(2t)\,dt$$

$$= -\frac{t}{2}\cot(2t) + \frac{1}{4}\ln|\sin(2t)| + C$$

(b) $\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx = I$

$u = \ln x$, $du = \frac{1}{x}dx$

$dv = \frac{dx}{\sqrt{x}}$, $V = 2\sqrt{x}$

$$I = \left[2\sqrt{x}\ln x\right]_1^4 - \int_1^4 \frac{2\sqrt{x}}{x}\,dx$$

$$= 4\ln 4 - \int_1^4 \frac{2}{\sqrt{x}}\,dx$$

$$= 8\ln 2 - \left[4\sqrt{x}\right]_1^4$$

$$= 8\ln 2 - [8 - 4]$$

$$= 8\ln 2 - 4$$

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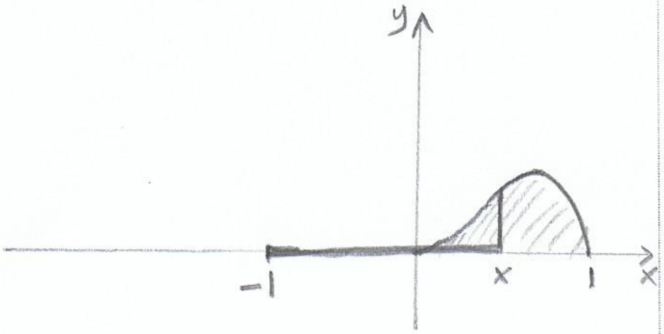
Quiz **3B** (Duration: 25 Minutes)

Name: Solution ID #: ____________ Section #: ____

Q1: (4 *points*) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2 - x^3$ and the line $y = 0$ about the line $x = -1$.

Intersection: $x^2 - x^3 = 0 \Rightarrow x^2(1-x) = 0$

$x = 0$ or $x = 1$



Circumference: $2\pi(x+1)$

Height: $x^2 - x^3$

$$V = \int_0^1 2\pi(x+1)(x^2 - x^3)\,dx$$

$$= 2\pi \int_0^1 (x^2 - x^4)\,dx$$

$$= 2\pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{4\pi}{15}$$

Q2: (2 *points*) Find the average value of $f(x) = 3x^2 + \sqrt{x}$ on the interval $[1, 2]$.

$$f_{ave} = \frac{1}{2-1}\int_1^2 (3x^2 + \sqrt{x})\,dx$$

$$= \left[x^3 + \frac{2}{3}x^{3/2}\right]_1^2$$

$$= 8 - 1 + \frac{2}{3}(2\sqrt{2} - 1)$$

$$= 7 + \frac{4\sqrt{2}}{3} - \frac{2}{3}$$

$$= \frac{1}{3}\left[19 + 4\sqrt{2}\right]$$

Q3: (4 *points*) Evaluate the integral

(a) $\int t\sec^2(2t)\,dt = I$

$u = t$, $du = dt$

$dv = \sec^2(2t)\,dt$

$v = \frac{1}{2}\tan(2t)$

$$I = \frac{t}{2}\tan(2t) - \int \frac{1}{2}\tan(2t)\,dt$$

$$= \frac{t}{2}\tan(2t) - \frac{1}{4}\ln|\sec(2t)| + C$$

$$= \frac{t}{2}\tan(2t) + \frac{1}{4}\ln|\cos(2t)| + C$$

(b) $\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx = I$

$u = \ln x$, $du = \frac{1}{x}\,dx$

$dv = \frac{dx}{\sqrt{x}}$, $V = 2\sqrt{x}$

$$I = \left[2\sqrt{x}\ln x\right]_1^4 - \int_1^4 \frac{2\sqrt{x}}{x}\,dx$$

$$= 4\ln 4 - \int_1^4 \frac{2}{\sqrt{x}}\,dx$$

$$= 8\ln 2 - (4\sqrt{x})\Big|_1^4$$

$$= 8\ln 2 - 4$$

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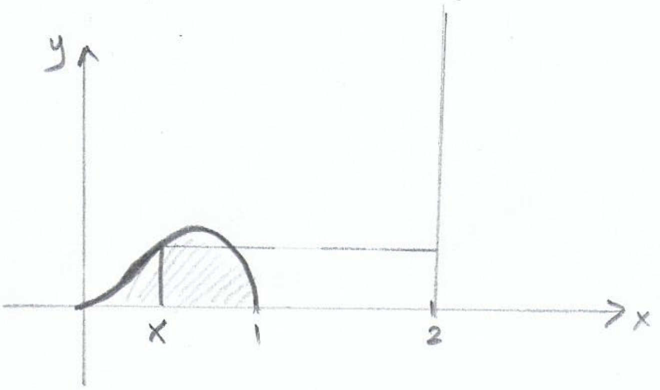
Quiz **3C** (Duration: 25 Minutes)

Name: Solution ID #: ______ Section #: ______

Q1: (4 *points*) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2 - x^3$ and the line $y = 0$ about the line $x = 2$.

Intersection : $x^2 - x^3 = 0 \Rightarrow x^2(1-x) = 0$

$x = 0$ or $x = 1$



Circumference : $2\pi(2-x)$

Height : $x^2 - x^3$

$V = \int_0^1 2\pi(2-x)(x^2 - x^3)\,dx$

$= 2\pi \int_0^1 (2x^2 - 3x^3 + x^4)\,dx$

$= 2\pi \left(\frac{2}{3} - \frac{3}{4} + \frac{1}{5}\right)$

$= \frac{7\pi}{30}$

Q2: (2 *points*) Find the average value of $f(x) = x + \frac{1}{\sqrt{x}}$ on the interval $[1, 4]$.

$f_{ave} = \frac{1}{4-1} \int_1^4 \left(x + \frac{1}{\sqrt{x}}\right) dx$

$= \frac{1}{3} \left(\frac{x^2}{2} + 2\sqrt{x}\right)\Big|_1^4$

$= \frac{1}{3}\left(\frac{16-1}{2} + 4 - 2\right)$

$= \frac{1}{3}\left(\frac{15}{2} + 2\right)$

$= \frac{19}{6}$

Q3: (4 *points*) Evaluate the integral

(a) $\int t \csc^2(2t)\,dt = I$

$u = t$, $du = dt$

$dv = \csc^2(2t)\,dt$

$v = -\frac{1}{2}\cot(2t)$

$I = -\frac{t}{2}\cot(2t) + \int \frac{1}{2}\cot(2t)\,dt$

$= -\frac{t}{2}\cot(2t) + \frac{1}{4}\ln|\sin 2t|$

$+ C$

(b) $\int_1^4 \frac{\ln x}{2\sqrt{x}}\,dx = I$

$u = \ln x$, $du = \frac{1}{x}dx$

$dv = \frac{dx}{2\sqrt{x}}$, $v = \sqrt{x}$

$I = \sqrt{x}\ln x\Big]_1^4 - \int_1^4 \frac{\sqrt{x}}{x}\,dx$

$= 2\ln 4 - \int_1^4 \frac{1}{\sqrt{x}}\,dx$

$= 2\ln 4 - \left[2\sqrt{x}\right]_1^4$

$= 4\ln 2 - 2$