KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **2A** (Duration: 25 Minutes)

Name: Solution ID #: ______ Section #: ______

Question 1: (5 *points*) Evaluate the integrals

(a) $\int \frac{1+\cos^2 x}{\cos^2 x}\,dx$

$= \int \left(\frac{1}{\cos^2 x} + 1\right) dx$

$= \int (\sec^2 x + 1)\, dx$

$= \tan x + x + C$

b) $\int_{-5}^{1} (x+2)^{-4}\left[\sin(x+2) + (x+2)\right] dx$

Let $u = x+2$, $du = dx$

$x = -5 \Rightarrow u = -3$

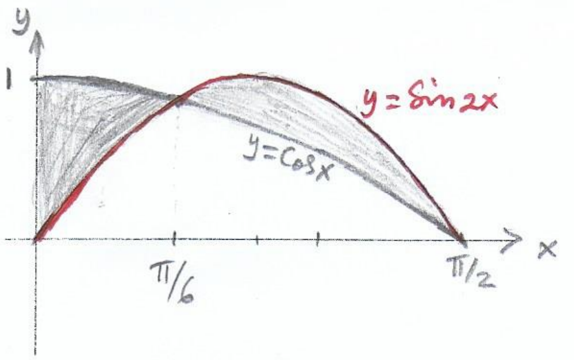
$x = 1 \Rightarrow u = 3$

$= \int_{-3}^{3} u^{-4}(\sin u + u)\, du$

$= 0$

(The integrand is an odd function!)

Question 2: (5 *points*) **Sketch** the region enclosed by the curves $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$ and **find** its area.



Area $= \int_0^{\pi/2} |\cos x - \sin 2x|\, dx$

$= \int_0^{\pi/6} (\cos x - \sin 2x)\, dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\, dx$

$= \left[\sin x + \frac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$

$= \left[\frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \left[\frac{1}{2} - 1 - \left(-\frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}\right)\right]$

$= \frac{1}{2}$

Intersection:

$\cos x = \sin 2x$

$\cos x = 2\sin x \cos x$

$\cos x(1 - 2\sin x) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$

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Quiz **2B** (Duration: 25 Minutes)

Name: Solution ID #: ____________ Section #: ____

Question 1: (6 *points*) Evaluate the integrals

(a) $\int \frac{1+\sin^2 x}{\sin^2 x}\,dx$

$= \int \left(\frac{1}{\sin^2 x} + 1\right) dx$

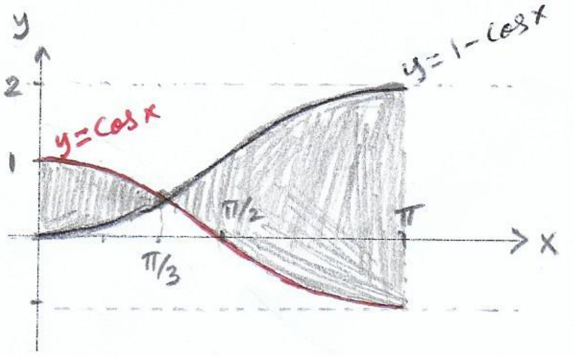
$= \int (\csc^2 x + 1)\, dx$

$= -\cot x + x + C$

(b) $\int_{-5}^{1} (x+2)^{-4} \left[\sin(x+2) + (x+2)\right] dx$

See Quiz 2A (Page 1)

Question 2: (4 *points*) **Sketch** the region enclosed by the curves $y = \cos x$, $y = 1 - \cos x$, $0 \le x \le \pi$, and **find** its area.



Intersection:

$\cos x = 1 - \cos x$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \pi/3$

$$\text{Area} = \int_0^{\pi} |\cos x - (1 - \cos x)|\, dx$$

$$= \int_0^{\pi/3} [\cos x - (1 - \cos x)]\, dx + \int_{\pi/3}^{\pi} [(1 - \cos x) - \cos x]\, dx$$

$$= \int_0^{\pi/3} (2\cos x - 1)\, dx + \int_{\pi/3}^{\pi} (1 - 2\cos x)\, dx$$

$$= \left[2\sin x - x\right]_0^{\pi/3} + \left[x - 2\sin x\right]_{\pi/3}^{\pi}$$

$$= \left(2\left(\tfrac{\sqrt{3}}{2}\right) - \pi/3\right) - 0 + \pi - \left(\tfrac{\pi}{3} - 2\left(\tfrac{\sqrt{3}}{2}\right)\right)$$

$$= 2\sqrt{3} + \frac{\pi}{3}$$

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MATH102 – Term 172
Quiz 2C (Duration: 25 Minutes)

Name: Solution ID #: ______ Section #: ______

Question 1: (5 *points*) Evaluate the integrals

(a) $\int \frac{1+\cos^2 x}{\cos^2 x}\, dx$

$$= \int \left(\frac{1}{\cos^2 x} + 1\right) dx$$

$$= \int (\sec^2 x + 1)\, dx$$

$$= \tan x + x + C$$

(b) $\int_{-2}^{2} \frac{x^2+\sin x}{1+x^2}\, dx$

$$= \int_{-2}^{2} \frac{x^2}{1+x^2}\, dx + \int_{-2}^{2} \frac{\sin x}{1+x^2}\, dx$$

$$= \int_{-2}^{2} \frac{x^2}{1+x^2}\, dx + 0$$

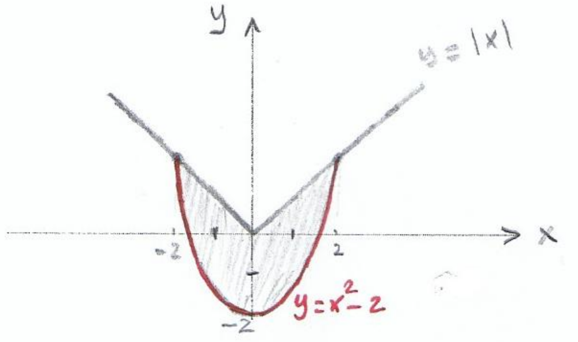
$$= \int_{-2}^{2} \frac{x^2+1-1}{1+x^2}\, dx$$

$$= \int_{-2}^{2} \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \left[x - \tan^{-1} x\right]_{-2}^{2}$$

$$= 4 - 2\tan^{-1}(2)$$

Question 2: (5 *points*) **Sketch** the region enclosed by the curves $y = |x|$, $y = x^2 - 2$, and **find** its area.



Intersection

$x > 0$,
$$x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2$$

$x < 0$,
$$-x = x^2 - 2 \Rightarrow x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2$$

$$\text{Area} = 2\int_0^2 [x - (x^2 - 2)]\, dx$$

$$= 2\left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right)_0^2$$

$$= 2\left(2 - \frac{8}{3} + 4\right)$$

$$= \frac{20}{3}$$