KFUPM – Department of Mathematics & Statistics
MATH102 – Term 172
Quiz **1A** (Duration: 25 Minutes)

Name: ______________________ ID #: ______________________ Section #:_____

Question 1: (7 *points*)

(a) Let $\boldsymbol{A}$ be the area of the region that lies under the graph of $f(x) = (2x-3)^2$ between $x = 1$ and $x = 4$. Estimate the area $\boldsymbol{A}$ by taking the sample points to be midpoints and using three subintervals.

$\Delta x = \frac{4-1}{3} = 1$

The subintervals are

$[1,2]$, $[2,3]$, $[3,4]$

using midpoints :

$A \approx \Delta x \left[f(\tfrac{3}{2}) + f(\tfrac{5}{2}) + f(\tfrac{7}{2}) \right]$

$= 0 + 4 + 16$

$= 20$

(b) Express the limit as a definite integral

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2} \left(\frac{1}{1+\left(\frac{2i}{n}\right)^2} \right)$$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n} \left[\frac{i/n}{1+4\left(\frac{i}{n}\right)^2} \right]$

$\Delta x = \frac{1}{n}$, $x_i = \frac{i}{n}$

$a = x_0 = 0$, $b = x_n = 1$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\frac{x_i}{1+4x_i^2} \right) \Delta x$

$= \int_0^1 \frac{x}{1+4x^2}\, dx$

Question 2: (3 *points*) Find

$$\frac{d}{dx} \int_{\cos x}^{\sin x} (1 - t^2)\, dt$$

$= (1-\sin^2 x) \frac{d}{dx}(\sin x)$
$\quad - (1-\cos^2 x) \frac{d}{dx}(\cos x)$

$= (1-\sin^2 x)(\cos x)$
$\quad + (1-\cos^2 x)(\sin x)$

$= (\cos^2 x)\cos x + \sin^2 x \sin x$

$= \cos^3 x + \sin^3 x$

KFUPM – Department of Mathematics & Statistics

MATH102 – Term 172

Quiz **1B** (Duration: 25 Minutes)

Name: ______________________ ID #: ______________________ Section #:_____

Question 1: (7 *points*)

(a) Let A be the area of the region that lies under the graph of $f(x) = (2x-3)^2$ between $x = 1$ and $x = 4$. Estimate the area A by taking the sample points to be midpoints and using three subintervals.

$\Delta x = \frac{4-1}{3} = 1$

The Subintervals are

$[1,2],\ [2,3],\ [3,4]$

using midpoints :

$A \approx \Delta x \left[f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) \right]$

$= 0 + 4 + 6$

$= 20$

(b) Express the limit as a definite integral

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n^2} \left(\frac{1}{1 + \left(\frac{i}{n}\right)^2} \right)$$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n} \left[\frac{2i/n}{1+(i/n)^2} \right]$

$\Delta x = \frac{1}{n}, \quad x_i = i/n$

$a = x_0 = 0, \quad b = x_n = 1$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2x_i}{1+x_i^2} \Delta x$

$= \int_0^1 \frac{2x}{1+x^2}\, dx$

Question 2: (3 *points*) Find

$$\frac{d}{dx} \int_{\sin x}^{\sec x} (t^2 - 1)\, dt$$

$= (\sec^2 x - 1) \frac{d}{dx}(\sec x) - (\sin^2 x - 1) \frac{d}{dx}(\sin x)$

$= (\sec^2 x - 1)(\sec x \tan x) - (\sin^2 x - 1)(\cos x)$

$= (\tan^2 x) \sec x \tan x + \cos^2 x \cos x$

$= \tan^3 x \sec x + \cos^3 x$