Math 102 Major Quiz 2

Name: Dr. Jamal **ID#** :) **Section#** Fun! **Serial#** :)

Question 1:

The volume of the solid generated by rotating the region bounded by the curves $y = -x^2 - x$, $y = 0$ about $x = 1$, equals to

$y = -x(x+1)$

(a) $\boxed{\frac{\pi}{2}}$

(b) $\frac{\pi}{4}$

(c) 2π

(d) $\frac{\pi}{3}$

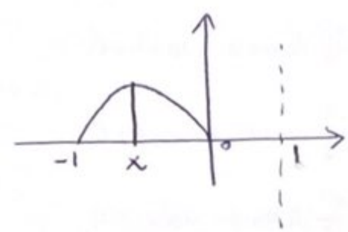
(e) 6π

$$V = 2\pi \int_{-1}^{0} -(1-x)\cdot x(1+x)\,dx$$

$$= -2\pi \int_{-1}^{0} (x - x^3)\,dx$$

$$= -2\pi \left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]\Big|_{-1}^{0}$$

$$= 2\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{2}.$$



$r = 1 - x$

$h = -x(x+1)$

Question 2:

The sum of all numbers c such that $f(c)$ equals the average value of the function $f(x) = 3x^2 - x + 4$ over the interval $[-1, 1]$ is

(a) $\boxed{\frac{1}{3}}$

(b) $\frac{1}{6}$

(c) $\frac{1}{2}$

(d) 2

(e) 1

$$f_{ave} = \frac{1}{2}\int_{-1}^{1}(3x^2 - x + 4)\,dx = 3c^2 - c + 4$$

$$\Rightarrow \frac{1}{2}\left(x^3 - \frac{1}{2}x^2 + 4x\right)\Big|_{-1}^{1} = 3c^2 - c + 4$$

$$\frac{1}{2}\left[\left(1 - \frac{1}{2} + 4\right) - \left(-1 - \frac{1}{2} - 4\right)\right] = 3c^2 - c + 4$$

$$5 = 3c^2 - c + 4$$

$$\Rightarrow 3c^2 - c - 1 = 0$$

$$c = \frac{1 \pm \sqrt{1 + 12}}{6} \Rightarrow c_1 + c_2 = \frac{1}{3}.$$

Question 3: $\int e^{2x} \sin(x)\, dx =$

$\boxed{\text{(a)}}$ $\frac{e^{2x}}{5}(2\sin x - \cos x) + C$

(b) $\frac{e^{2x}}{5}(3\sin x + \cos x) + C$

(c) $\frac{e^{2x}}{3}(2\cos x - \sin x) + C$

(d) $\frac{e^{2x}}{2}(3\sin x + \cos x) + C$

(e) $\frac{e^{2x}}{7}(5\sin x - \cos x) + C$

$$= -e^{2x}\cos x + 2\int e^{2x}\cos x\, dx$$

$$= -e^{2x}\cos x + 2\left[e^{2x}\sin x - 2\int e^{2x}\sin x\, dx\right]$$

$u = e^{2x}$ $dv = \sin x\, dx$

$du = 2e^{2x}\, dx$ $v = -\cos x$

$u = e^{2x}$ $dv = \cos x\, dx$

$du = 2e^{2x}\, dx$ $v = \sin x\, dx$

$$\Rightarrow 5\int e^{2x}\sin x\, dx = -e^{2x}\cos x + 2e^{2x}\sin x$$

$$\Rightarrow \int e^{2x}\sin x\, dx = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$$

Question 4:

If $I = \int_1^e \ln^2(x)\, dx$, then I is equal to

$\boxed{\text{(a)}}$ $e-2$

(b) $e+2$

(c) e

(d) 2

(e) $2e$

$$= x\ln^2 x \Big|_1^e - \int_1^e 2\ln x\, dx$$

$$= e - \left[2x\ln x \Big|_1^e - \int_1^e 2\, dx\right]$$

$$= e - 2e + 2(e-1)$$

$$= e - 2$$

$u = \ln^2 x$ $dv = dx$

$du = 2\ln x \cdot \frac{1}{x}\, dx$ $v = x$

$u = 2\ln x$ $dv = dx$

$du = \frac{2}{x}\, dx$ $v = x$