Name: ID: Section: Sr:

Question 1:

$$\frac{d}{dx}\left[\int_{\sqrt{x}}^{2}\cos(t^2)\,dt\right]= \frac{d}{dx}\left[-\int_{2}^{\sqrt{x}}\cos(t^2)\,dt\right]$$

$$= -\cos(\sqrt{x})^2\cdot\frac{1}{2\sqrt{x}}$$

$$= -\cos x\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{-\cos x}{2\sqrt{x}}$$

(a) $\frac{\cos x}{\sqrt{x}}$

(b) $\cos 4-\cos x$

(c) $\sin 4-\sin x$

(d) $\frac{\sin 4}{4}-\frac{\sin x}{2\sqrt{x}}$

$\boxed{\text{(e) } -\frac{\cos x}{2\sqrt{x}}}$

Question 2:

$$\lim_{n\to+\infty}\frac{1}{n}\left(\sqrt[3]{\frac{1}{n}}+\sqrt[3]{\frac{2}{n}}+\sqrt[3]{\frac{3}{n}}+\ldots+\sqrt[3]{\frac{n}{n}}\right)=$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\sqrt[3]{\frac{i}{n}}\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\sqrt[3]{x_i}\;\Delta x$$

$$= \int_0^1 x^{1/3}\,dx$$

$$= \frac{3}{4}x^{4/3}\Big|_0^1$$

$$= \frac{3}{4}-0$$

$$= \frac{3}{4}$$

Let $\boxed{x_i=\frac{i}{n}}$

$\Rightarrow \frac{i}{n}=a+i\Delta x$

$\Rightarrow \boxed{a=0},\ \boxed{\Delta x=\frac{1}{n}}$

$\Rightarrow \frac{1}{n}=\frac{b-a}{n}$

$\Rightarrow b-a=1 \Rightarrow \boxed{b=1}$

$\boxed{\text{(a) } \frac{3}{4}}$

(b) 0

(c) $\sqrt[3]{4}$

(d) 1

(e) $\frac{3}{2}$

Question 3:

$y^2-4=x$ $\qquad$ $x=2-y^2$

The area of the region bounded by the curves $y^2-x=4$ and $y^2+x=2$ is equal to

(a) 4

$$\begin{cases} x=y^2-4 \\ x=2-y^2 \end{cases} \Rightarrow y^2-4=2-y^2$$

$$\Rightarrow 2y^2=6$$

(b) 6

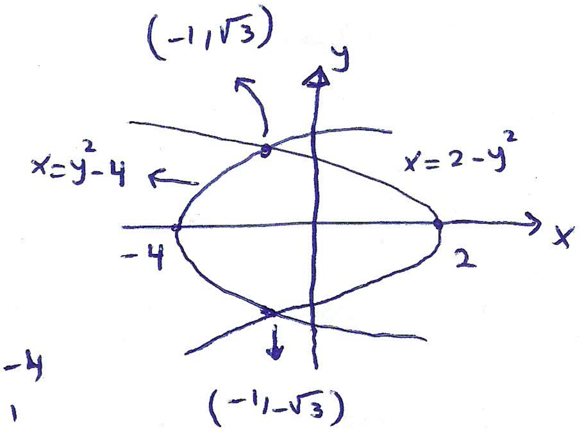
$$\Rightarrow y^2=3$$

$$\Rightarrow y=-\sqrt{3} \quad \Big| \quad y=\sqrt{3}$$

$$x=3-4=-1 \quad \Big| \quad x=3-4=-1$$

(c) $4\sqrt{3}$

The points : $(-1,-\sqrt{3})$ | $(-1,\sqrt{3})$



$\boxed{\text{(d) } 8\sqrt{3}}$

$$\Rightarrow A=\int_{-\sqrt{3}}^{\sqrt{3}} 2-y^2-(y^2-4)\,dy=\int_{-\sqrt{3}}^{\sqrt{3}}[-2y^2+6]\,dy$$

(e) 3

$$=2\int_0^{\sqrt{3}}[-2y^2+6]\,dy=2\left[-\frac{2y^3}{3}+6y\right]_0^{\sqrt{3}}$$

$$=2\left[-2\sqrt{3}+6\sqrt{3}-0\right]=8\sqrt{3}$$

Question 4: $I=\displaystyle\int_1^4 \frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}\,dx=$

let $u=e^{\sqrt{x}} \Rightarrow du=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

(a) $4\cos e$

$$\Rightarrow 2\,du=\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

(b) $2\left(\cos e-\cos e^2\right)$

$x=1 \Rightarrow u=e^{\sqrt{1}}=e$

$x=4 \Rightarrow u=e^{\sqrt{4}}=e^2$

$\boxed{\text{(c) } 2\left(\sin e^2-\sin e\right)}$

$$I=2\int_e^{e^2}\cos u\,du$$

(d) $\frac{1}{2}\left(\cos e-\cos e^2\right)$

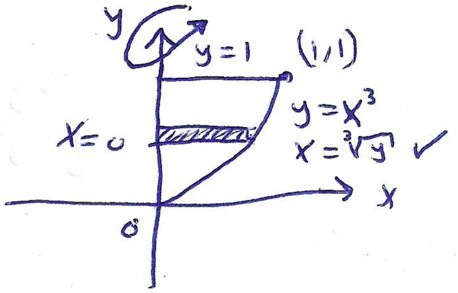
$$=2\left[\sin u\right]_e^{e^2}$$

(e) $4\sin e$

$$=2\left[\sin e^2-\sin e\right]$$

Question 5:

The volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 1$, and $x = 0$ about the y-axis is equal to



(a) $\frac{3\pi}{7}$

(b) $\frac{\pi}{5}$

(c) $\frac{3\pi}{4}$

(d) $\frac{2\pi}{3}$

$\boxed{\text{(e) } \frac{3\pi}{5}}$

the cross-section will be a disk with $r = \sqrt[3]{y}$

$$V = \pi \int_0^1 (\sqrt[3]{y})^2 \, dy$$

$$= \pi \int_0^1 y^{2/3} \, dy$$

$$\pi \left[\frac{3}{5} y^{5/3} \right]_0^1$$

$$= \pi \left[\frac{3}{5} - 0 \right]$$

$$= \frac{3\pi}{5}$$

Question 6:

A particle moves along a line so that its velocity at time t is $v(t) = \sin t$ (measured in meters per second). The distance traveled by the particle during the time period $0 \le t \le \frac{3\pi}{2}$ is equal to

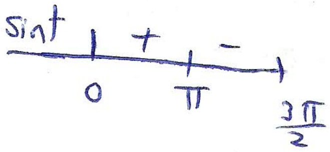
$\boxed{\text{(a) 3 meters}}$

(b) 2 meters

(c) 1 meters

(d) $\frac{3}{2}$ meters

(e) $\frac{1}{2}$ meters



$$\text{distance} = \int_0^{3\pi/2} |\sin t| \, dt$$

$$= \int_0^{\pi} \sin t \, dt + \int_{\pi}^{3\pi/2} -\sin t \, dt$$

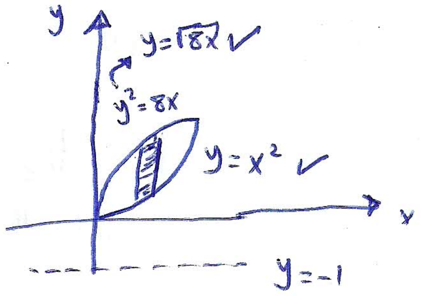
$$= -\cos t \Big]_0^{\pi} + \cos t \Big]_{\pi}^{3\pi/2}$$

$$= -(-1-1) + 0-(-1)$$

$$= 2 + 1 = 3 \text{ m}$$

Question 7:

The volume of the solid generated by revolving the region bounded by the parabolas $y = x^2$ and $y^2 = 8x$ about the line $y = -1$ is given by



(a) $\pi \int_0^2 (8x - x^4)\, dx$

$\begin{cases} y = x^2 \\ y^2 = 8x \end{cases} \Rightarrow x^4 = 8x$

$\Rightarrow x^4 - 8x = 0$

$x(x^3 - 8) = 0$

(b) $\boxed{\pi \int_0^2 \left[(\sqrt{8x} + 1)^2 - (x^2 + 1)^2 \right] dx}$

$x = 0$, $y = 0$ | $x = 2$, $y = 4$

(c) $\pi \int_0^{16} \left[(\sqrt{y} + 1)^2 - \left(\frac{1}{8} y^2 + 1 \right)^2 \right] dy$

The Point: $(0,0)$ $(2,4)$

$r_{out} = \sqrt{8x} - (-1) = \sqrt{8x} + 1$

(d) $\pi \int_0^{16} \left[(\sqrt{y} - 1)^2 - \left(\frac{1}{8} y^2 - 1 \right)^2 \right] dy$

$r_{in} = x^2 - (-1) = x^2 + 1$

(e) $\pi \int_0^2 (\sqrt{8x} - x^2)^2\, dx$

$V = \pi \int_0^2 (\sqrt{8x} + 1)^2 - (x^2 + 1)\, dx$

Question 8:

If f is continuous on $[0,1]$ and $\int_0^1 f(x)\, dx = 2$, then $\int_0^1 f(1-x)\, dx = I$,

let $u = 1 - x \Rightarrow du = -dx$

$\Rightarrow \boxed{-du = dx}$

(a) -2

$x = 0 \Rightarrow u = 1 - 0 = 1$

$x = 1 \Rightarrow u = 1 - 1 = 0$

(b) 1

$\Rightarrow I = \int_1^0 f(u)\, (-du)$

(c) 0

$= -\int_1^0 f(u)\, du$

(d) -1

$= \int_0^1 f(u)\, du = 2$

(e) $\boxed{2}$