1. The average value of $f(x) = \dfrac{x}{(x^2+1)^3}$ from 1 to 3 is

(a) 0.03

(b) 0.04

(c) 0.05

(d) 0.01

(e) 0.02

2. Using the method of partial fractions, if

$$\frac{x}{(x-1)(x-2)(x+1)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1},$$

then $2A + 3B - 6C =$

(a) 2

(b) 0

(c) -2

(d) 4

(e) 6

3. Using the method of cylindrical shells, the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 0$ and $y = 1$ about the line $y = 2$ is given by

(a) $v = \int_0^1 2\,\pi\,(2y^2 - y^3)\,dy$

(b) $v = \int_0^1 2\,\pi\,y^3\,dy$

(c) $v = \int_0^1 2\,\pi\,(x-2)\,\sqrt{x}\,dx$

(d) $v = \int_0^2 2\,\pi\,(y - y^2)\,dy$

(e) $v = \int_0^1 2\,\pi\,(2-y)\,y\,dy$

4. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $-\pi < x < \pi$, we obtain $\int \frac{dx}{1+\cos x} =$

(a) $\tan\left(\frac{x}{2}\right) + C$

(b) $\sin\left(\frac{x}{2}\right) + C$

(c) $\tan\left(\frac{x}{4}\right) + C$

(d) $\sin\left(\frac{x}{4}\right) + C$

(e) $-\tan\left(\frac{x}{3}\right) + C$

5. $\displaystyle\int \tan^3 x \ \sec^5 x \, dx =$

(a) $\dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5} + C$

(b) $\dfrac{\sec^7 x}{7} + \dfrac{\sec^3 x}{3} + C$

(c) $\dfrac{\sec^3 x}{3} - \sec x + C$

(d) $\dfrac{\tan^4 x}{4} + \dfrac{\tan^2 x}{2} + C$

(e) $\dfrac{\tan^7 x}{7} - \dfrac{\sec^5 x}{5} + C$

6. $\displaystyle\int_0^{\pi/2} \cos(3x) \ \cos(2x) \, dx =$

(a) $\dfrac{3}{5}$

(b) $\dfrac{1}{5}$

(c) $\dfrac{2}{5}$

(d) $\dfrac{4}{5}$

(e) $\dfrac{7}{5}$

7. $\displaystyle\int_0^\infty \frac{16\,\tan^{-1}x}{1+x^2}\,dx =$

(a) $2\,\pi^2$

(b) π^2

(c) $4\,\pi^2$

(d) $6\,\pi^2$

(e) ∞

8. Using the method of cylindrical shells, the volume of the solid generated by rotating the region bounded by the curves $y = 4x - x^2$ and $y = x$ about the y-axis is given by

(a) $v = 2\,\pi \displaystyle\int_0^3 (3x^2 - x^3)\,dx$

(b) $v = \pi \displaystyle\int_0^3 (4x^2 - x^3)\,dx$

(c) $v = 2\,\pi \displaystyle\int_0^3 (2x^2 - x^3)\,dx$

(d) $v = \pi \displaystyle\int_0^3 (3x^2 + x^3)\,dx$

(e) $v = 2\,\pi \displaystyle\int_0^3 (6x^2 - x^3)\,dx$

9. $\int_0^{\pi/2} (2 - \sin\,\theta)^2\, d\theta =$

(a) $\dfrac{9\,\pi}{4} - 4$

(b) $\dfrac{\pi}{4} + 4$

(c) $\dfrac{3\,\pi}{4} - 1$

(d) $\dfrac{3\,\pi}{4} + 1$

(e) $9\,\pi$

10. Using trigonometric substitution $\displaystyle\int \frac{dx}{\sqrt{x^2+9}} =$

(a) $\ln\left(x + \sqrt{x^2+9}\right) + c$

(b) $\ln\left(x - \sqrt{x^2+9}\right) + c$

(c) $\ln\left(x + 2\sqrt{x^2+9}\right) + c$

(d) $\ln\left(2x + \sqrt{x^2+9}\right) + c$

(e) $\ln\left(2x - \sqrt{x^2+9}\right) + c$

11. $\int_0^{\pi} (3t+2)\cos\left(\frac{t}{2}\right)\,dt =$

(a) $6\pi - 8$

(b) $8\pi - 6$

(c) $6\pi + 6$

(d) 6π

(e) $8 - \pi$

12. $\int e^x \tan^{-1}(e^x)\,dx =$

(a) $e^x \tan^{-1}(e^x) - \frac{1}{2}\ln(1+e^{2x}) + c$

(b) $e^x \tan^{-1}(e^x) - \ln(1+e^{2x}) + c$

(c) $e^x \tan^{-1}(e^x) + \ln(1-e^{2x}) + c$

(d) $e^x \tan^{-1}(e^{2x}) - \ln(1+e^{2x}) + c$

(e) $e^x \tan^{-1}(e^{2x}) + \ln(1-e^x) + c$

13. The length of the arc of the curve

$$x = \int_0^y \sqrt{-1+\sec^4 t}\, dt, \quad \frac{-\pi}{4} \le y \le \frac{\pi}{4}$$

is equal to

(a) 2

(b) 4

(c) 6

(d) 8

(e) 10

14. $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\, dx =$

(a) $2\ln|x| - \frac{1}{2}\ln(x^2+3) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

(b) $\ln|x| - \ln(x^2+3) + \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

(c) $2\ln|x| + 3\ln(x^2+3) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

(d) $\ln|x| - \frac{1}{2}\ln(x^2+3) + \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

(e) $3\ln|x| + \frac{1}{3}\ln(x^2+3) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$

15. $\int x^5\, e^{-x^3}\, dx =$

(a) $-\frac{1}{3}(x^3+1)\, e^{-x^3} + c$

(b) $-(x^3+1)\, e^{-x^3} + c$

(c) $-\frac{1}{3}(x^3-1)\, e^{-x^3} + c$

(d) $\frac{1}{3}(x^3+1)\, e^{x^3} + c$

(e) $2\,(x^3+1)\, e^{x^3} + c$

16. The length of the arc of the curve

$$y = \sin^{-1} x + \sqrt{1-x^2},\ \frac{-1}{2} \le x \le 0$$

is equal to

(a) $2\sqrt{2} - 2$

(b) $3\sqrt{2} - 2$

(c) $\sqrt{2} - 2$

(d) $5\sqrt{2} - 2$

(e) $4\sqrt{2} - 2$

17. $\displaystyle\int \frac{dx}{x + x\sqrt{x}} =$

(a) $2 \ln\left(\dfrac{\sqrt{x}}{1+\sqrt{x}}\right) + c$

(b) $\ln\left(\dfrac{\sqrt{x}}{1+\sqrt{x}}\right) + c$

(c) $4 \ln\left(\dfrac{\sqrt{x}}{1-\sqrt{x}}\right) + c$

(d) $2 \ln\left(\dfrac{1+\sqrt{x}}{\sqrt{x}}\right) + c$

(e) $4 \ln\left(\dfrac{1+\sqrt{x}}{\sqrt{x}}\right) + c$

18. The improper integral $\displaystyle\int_{-1}^{4} \frac{dx}{\sqrt{|x|}}$ is

(a) convergent and equals 6

(b) convergent and equals 3

(c) convergent and equals 8

(d) convergent and equals 2

(e) divergent

19. If $\int \sqrt{5+4x-x^2}\,dx = A\sin^{-1}\left(\frac{(x-2)}{3}\right) + B(x-2)\sqrt{5+4x-x^2} + C$, then $A - B =$

(a) 4

(b) 6

(c) 8

(d) 10

(e) 12

20. If the average value of $f(x) = 3x^2 - 2ax - b$ on $[a, b] (a \neq b)$ is $-\frac{1}{4}$, then $b =$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $-\frac{1}{4}$

(e) 0