1. $\lim_{x\to 1} \cos\left(\frac{1}{x-1}\right)$

$\lim_{x\to 1} \cos\left(\frac{1}{x-1}\right)$ does not exist

(a) does not exist

(b) is equal to 0

(c) is equal to 2

(d) is equal to 1

(e) is equal to -1

2. If $\lim_{x\to a} f(x) = L$, which one of the following statements is **necessarily TRUE**?

(a) $\lim_{x\to a^+} f(x) = L$

(b) f is continuous at a

(c) $f(a)$ does not exist

(d) $f(a) = L$

(e) $\lim_{x\to a^-} f(x)$ is not equal to L

If $\lim_{x\to a} f(x) = L$, then we necessarily have $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^-} f(x) = L$

3. An equation of the slant (oblique) asymptote for the graph of $f(x) = \dfrac{2x^2 - x + 1}{x + 1}$ is

(a) $y = 2x - 3$

(b) $y = 2x + 3$

(c) $y = 2x - 1$

(d) $y = 2x + 1$

(e) $y = 2$

$$\begin{array}{r|l} 2x^2 - x + 1 & x+1 \\ \cline{2-2} -\;\; 2x^2 + 2x & 2x - 3 \\ \hline -3x + 1 & \\ -\;\; -3x - 3 & \\ \hline 4 & \end{array}$$

Thus, $\dfrac{2x^2 - x + 1}{x+1} = 2x - 3 + \dfrac{4}{x+1}$

and $\lim\limits_{x \to \mp\infty} \left[\dfrac{2x^2 - x + 1}{x+1} - (2x - 3) \right] = 0.$

Hence, $y = 2x - 3$ is a slant asymptote

4. If $2x \le g(x) \le x^4 - x^2 + 2$, then $\lim\limits_{x \to 1} g(x)$

(a) is equal to 2

(b) is equal to 0

(c) is equal to 1

(d) does not exist

(e) is equal to -2

$\lim\limits_{x \to 1} 2x = 2$

and $\lim\limits_{x \to 1} (x^4 - x^2 + 2) = 2.$

Using Squeeze Theorem, we have $\lim\limits_{x \to 1} g(x) = 2$

5. If $y = \ln\left(\frac{1}{x}\right)$, then $y'' =$

$y = -\ln x$

$y' = -\frac{1}{x}$ and $y'' = \frac{1}{x^2}$

(a) $\frac{1}{x^2}$

(b) $\frac{-1}{x^2}$

(c) $\frac{1}{x}$

(d) $\frac{-1}{x}$

(e) $\frac{2}{x^3}$

6. Using Newton's method with initial approximation $x_1 = 2$, the second approximation x_2 to the root of the equation $x^3 - 2x - 5 = 0$ is

Let $f(x) = x^3 - 2x - 5$.

Then, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$.

$f'(x) = 3x^2 - 2$.

We have $f(2) = -1$, $f'(2) = 10$

So that, $x_2 = 2 + \frac{1}{10} = 2.1$

(a) 2.1

(b) 2.01

(c) 2.05

(d) 2.15

(e) 2.2

7. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, then the rate at which the diameter decreases when the diameter is 10 cm is
(Note: the surface area of a sphere of radius r is $S = 4\,\pi\,r^2$)

(a) $\frac{1}{20\,\pi}\,cm/min$

(b) $\frac{1}{30\,\pi}\,cm/min$

(c) $\frac{1}{10\,\pi}\,cm/min$

(d) $\frac{1}{50\,\pi}\,cm/min$

(e) $\frac{1}{40\,\pi}\,cm/min$

$S = 4\pi r^2$, $c = 2r$

$S = 4\pi\left(\frac{c}{2}\right)^2 = \pi c^2$

$\Rightarrow \frac{dS}{dt} = 2\pi c \frac{dc}{dt}$

and $\frac{dc}{dt} = \frac{1}{2\pi c}\frac{dS}{dt}$

$= \frac{-1}{20\pi}$ cm/min

8. If $\sinh x = -\sqrt{3}$, then $\operatorname{sech} x =$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) 3

(d) -4

(e) $\frac{\sqrt{2}}{2}$

$\cosh x > 0$, for all x.

$\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \cosh x = \sqrt{1+3} = 2$

and $\operatorname{sech} x = \frac{1}{2}$

9. $\lim_{x\to\infty} \tanh x - \lim_{x\to-\infty} \tanh x =$

$\lim_{x\to\infty} \tanh x = \lim_{x\to\infty} \frac{1-e^{-2x}}{1+e^{-2x}} = 1$

(a) 2

$\lim_{x\to-\infty} \tanh x = \lim_{x\to-\infty} \frac{e^{2x}-1}{e^{2x}+1} = -1$

(b) 0

(c) -1

$\Rightarrow \lim_{x\to\infty} \tanh x - \lim_{x\to-\infty} \tanh x = 2$

(d) 1

(e) 3

10. Using the linear approximation of the function $f(x) = \cos\left(x + \frac{\pi}{6}\right)$ at $x = 0$, we find that $\cos\left(\frac{\pi}{6} + 0.1\right) \approx$

$f(x) \simeq f(0) + x f'(0)$

(a) $\frac{\sqrt{3}}{2} - \frac{1}{20}$

$f'(x) = -\sin\left(x + \frac{\pi}{6}\right)$

(b) $\frac{1}{2} - \frac{\sqrt{3}}{20}$

$f(0) = \frac{\sqrt{3}}{2}$; $f'(0) = -\frac{1}{2}$

(c) $\frac{1}{2} + \frac{\sqrt{3}}{20}$

$\Rightarrow f(x) \simeq \frac{\sqrt{3}}{2} - \frac{x}{2}$

(d) $\frac{\sqrt{3}}{2} + \frac{1}{20}$

and $f(0.1) \simeq \frac{\sqrt{3}}{2} - \frac{1}{20}$

(e) $\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{20}$

11. $\lim_{x\to\infty}\left(\sqrt{x^2+2x+3}-\sqrt{x^2+1}\right) =$

(a) 1

(b) 2

(c) 3

(d) 0

(e) ∞

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x+3}-\sqrt{x^2+1}\right)=\lim_{x\to\infty}\frac{(x^2+2x+3)-(x^2+1)}{\sqrt{x^2+2x+3}+\sqrt{x^2+1}}$$

$$=\lim_{x\to\infty}\frac{2+\frac{2}{x}}{\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{1}{x^2}}}$$

$$=\frac{2}{2}=1$$

12. If $f''(x) = 3x + 2e^x$, $f'(0) = 3$ and $f(0) = 4$, then $f(x) =$

(a) $2e^x + \frac{1}{2}x^3 + x + 2$

(b) $2e^x + \frac{1}{2}x^3 + x$

(c) $\frac{1}{2}e^x + \frac{1}{2}x^3 + x + 2$

(d) $2e^x + \frac{3}{2}x^2 + 1$

(e) $2e^x + \frac{3}{2}x^3 + x + 2$

$f'(x) = \frac{3x^2}{2} + 2e^x + C_1$

$f'(0) = 3 \Rightarrow 2 + C_1 = 3,\ C_1 = 1$

$f'(x) = \frac{3}{2}x^2 + 2e^x + 1$

$\Rightarrow f(x) = \frac{x^3}{2} + 2e^x + x + C_2$

$f(0) = 4 \Rightarrow 2 + C_2 = 4,\ C_2 = 2$

$f(x) = \frac{x^3}{2} + 2e^x + x + 2$

13. If the function $f(x) = \begin{cases} \dfrac{8x + \sin 2x}{5x} & \text{if} \quad x \neq 0 \\ k & \text{if} \quad x = 0 \end{cases}$

is continuous everywhere, then $k =$

(a) 2

(b) 1

(c) 3

(d) 4

(e) 5

$\lim\limits_{x \to 0} \dfrac{8x + \sin 2x}{5x} = \lim\limits_{x \to 0} \dfrac{8 + 2\cos 2x}{5}$

$= \dfrac{10}{5} = 2$

$\Rightarrow k = 2$

14. The function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ is increasing on the interval(s)

(a) $(-1, 0)$ and $(2, \infty)$

(b) $(-\infty, -1)$ and $(0, 2)$

(c) $(-\infty, 2)$

(d) $(-1, 2)$

(e) $(0, \infty)$

$f'(x) = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$= 12x(x+1)(x-2)$

x	$-\infty$		-1		0		2		∞
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	

f is increasing on $(-1, 0)$ and on $(2, \infty)$

15. If $f(x) = \dfrac{x^2-4}{2x+5}$, then $f'(x) =$

$$f'(x) = \frac{2x(2x+5) - 2(x^2-4)}{(2x+5)^2}$$
$$= \frac{2x^2+10x+8}{(2x+5)^2}$$

(a) $\dfrac{2x^2+10\,x+8}{(2x+5)^2}$

(b) $\dfrac{2x^2+3\,x-4}{(2x+5)^2}$

(c) $\dfrac{4x^2+5\,x-2}{(2x+5)^2}$

(d) $\dfrac{x+3}{2x+5}$

(e) $\dfrac{5x+4}{(2x+5)^2}$

16. The absolute maximum M and absolute minimum m values of $f(x) = e^{-(x^2-2x)}$ on the interval $[0,3]$ are

$$f'(x) = -(2x-2)\,e^{-(x^2-2x)}$$
$$f'(x) = 0 \iff x = 1$$
$$\begin{cases} f(1) = e \\ f(0) = 1 \\ f(3) = e^{-3} \end{cases}$$
$$\Rightarrow M = e \text{ and } m = e^{-3}$$

(a) $M = e$ and $m = e^{-3}$

(b) $M = e$ and $m = 1$

(c) $M = 1$ and $m = e^{-3}$

(d) $M = 2e$ and $m = 1$

(e) $M = 3e$ and $m = e$

17. The edge of a cube was found to be $10\,cm$ with possible error in measurement of $0.1\,cm$. By using differentials, the maximum possible percentage error in computing the volume of the cube is

(a) $3\,\%$

(b) $0.1\,\%$

(c) $1\,\%$

(d) $4\,\%$

(e) $10\,\%$

$V = x^3, \quad dV = 3x^2\,dx$

$\Rightarrow \dfrac{dV}{V} = \dfrac{3x^2\,dx}{x^3} = 3\,\dfrac{dx}{x}$

$= 3 \cdot \dfrac{0.1}{10} = 3 \cdot \dfrac{1}{100}$

$= 3\,\%$

18. The position function of a particle moving in a straight line is given by $s(t) = t^2\,e^{-t}$, where t is measured in seconds and s in meters. Then, the total distance traveled during the time interval $0 \le t \le 3$ is

(a) $\dfrac{8e-9}{e^3}\ m$

(b) $9\,e^3\ m$

(c) $\dfrac{4e-9}{e^3}\ m$

(d) $\dfrac{6e-9}{e^3}\ m$

(e) $\dfrac{9-2e}{e^3}\ m$

$v(t) = (2t - t^2)\,e^{-t}$

t	0		2		∞
$v(t)$	0	$+$	0	$-$	

$t=3$

$t=0$ $\qquad$ $t=2$

Total distance $= |s(2) - s(0)| + |s(3) - s(2)|$

$s(0) = 0, \quad s(2) = 4e^{-2}, \quad s(3) = 9e^{-3}$

$\Rightarrow$ Total distance $= \dfrac{4}{e^2} + \dfrac{4}{e^2} - \dfrac{9}{e^3}$

$= \dfrac{8e-9}{e^3}\ m$

19. State why Rolle's theorem does not apply to $f(x) = 1 - x^{2/3}$ on the interval $[-1, 1]$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x} = \lim_{x\to 0^+}\left(-\frac{x^{\frac{2}{3}}}{x}\right)$$

$$= \lim_{x\to 0^+}\left(-\frac{1}{x^{1/3}}\right) = -\infty$$

$\Rightarrow$ f is not differentiable at $x=0$

(a) f is not differentiable at $x = 0$

(b) f is not continuous on $[-1, 1]$

(c) f is not defined on the whole interval

(d) $f(1)$ is not equal to $f(-1)$

(e) f is constant on $[-1, 1]$

20. The number c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x + \sin 2x$ on the interval $\left[0, \frac{\pi}{2}\right]$ is

$$f'(c) = \frac{f(\frac{\pi}{2}) - f(0)}{\pi/2}$$

$f(0) = 0$, $f(\pi/2) = \pi/2$

$f'(x) = 1 + 2\cos 2x$

$\Rightarrow$ $1 + 2\cos 2c = 1$

$\cos 2c = 0$, $2c = \frac{\pi}{2}$

$c = \frac{\pi}{4}$

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) 0

(e) $\frac{\pi}{6}$

21. The slope of the tangent line to the curve $\tan(x+y) = \tan^{-1}(xy) + 1$ at the point $\left(\frac{\pi}{4}, 0\right)$ is

(a) $\frac{8}{\pi - 8}$

(b) $\frac{8}{\pi}$

(c) $\pi - 8$

(d) $\frac{8}{8 - \pi}$

(e) $\frac{8}{8 + \pi}$

$$(1+y')\sec^2(x+y) = \frac{y + xy'}{1+(xy)^2}$$

$$\Rightarrow \left(1 + y'\left(\tfrac{\pi}{4}\right)\right) \underbrace{\sec^2\left(\tfrac{\pi}{4}\right)}_{=2} = \frac{\frac{\pi}{4}\, y'\left(\frac{\pi}{4}\right)}{1}$$

$$y'\left(\tfrac{\pi}{4}\right) = \frac{1}{\frac{\pi}{8} - 1} = \frac{8}{\pi - 8}$$

22. If $h(x) = f(g(x))$, $g(0) = 1$, $h'(0) = 4$ and $f'(1) = 2$, then $g'(0) =$

(a) 2

(b) 4

(c) 1

(d) 8

(e) 6

$$h'(x) = g'(x) \times f'(g(x))$$

$$\Rightarrow h'(0) = g'(0) \times f'(g(0))$$

$$4 = g'(0) \times \underbrace{f'(1)}_{=2}$$

$$g'(0) = 2$$

23. Let $f(x) = x^{4/3} + 4x^{1/3}$.
Which one of the following statements is **TRUE**?

(a) f has an inflection point at $(2, 6\sqrt[3]{2})$

(b) f has an inflection point at $(-1, -3)$

(c) f is concave upward on $(0, 2)$

(d) f is concave downward on $(4, \infty)$

(e) f is concave downward on $(-\infty, -1)$

$f'(x) = \frac{4}{3}\left(x^{1/3} + x^{-2/3}\right)$

$f''(x) = \frac{4}{9}\left(x^{-\frac{2}{3}} - 2x^{-5/3}\right)$

$= \frac{4}{9} x^{-5/3}(x-2)$

x	$-\infty$	0	2	∞
$f''(x)$	+	‖	− 0 +	

f has an inflection point at $(2, 6\sqrt[3]{2})$

24. If $y = e^{\tan^{-1} x}$, then $(1 + x^2)^2\, y'' + (1 + x^2)\, y' + 2xy =$

(a) $2y$

(b) $-3y$

(c) xy

(d) $x^2\, y$

(e) $-3xy$

$y' = \frac{1}{1+x^2} e^{\tan^{-1} x}$

$y'' = \frac{-2x}{(1+x^2)^2} e^{\tan^{-1} x} + \frac{1}{(1+x^2)^2} e^{\tan^{-1} x}$

$\Rightarrow (1+x^2)^2 y'' = (-2x+1)\, e^{\tan^{-1} x}$

$(1+x^2)\, y' = e^{\tan^{-1} x}$

$2xy = 2x\, e^{\tan^{-1} x}$

$(1+x^2)^2 y'' + (1+x^2) y' + 2xy = 2e^{\tan^{-1} x} = 2y$

25. $\lim_{x \to 0} (1-x)^{\cot\left(\frac{\pi x}{2}\right)} =$

(a) $e^{-\frac{2}{\pi}}$

(b) 1

(c) ∞

(d) e^{π}

(e) $e^{-3\pi}$

$$(1-x)^{\cot\left(\frac{\pi x}{2}\right)} = e^{\cot\left(\frac{\pi x}{2}\right)\ln(1-x)}$$

$$= e^{\cos\left(\frac{x\pi}{2}\right)\frac{\ln(1-x)}{\sin\left(\frac{\pi x}{2}\right)}}$$

$$\lim_{x\to 0} \cos\left(\frac{x\pi}{2}\right) = 1$$

and $\lim_{x\to 0} \frac{\ln(1-x)}{\sin\left(\frac{\pi x}{2}\right)} = \lim_{x\to 0} \frac{\frac{-1}{1-x}}{\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)}$

$$= -\frac{2}{\pi}$$

$$\Rightarrow \lim_{x\to 0} (1-x)^{\cot\left(\frac{\pi x}{2}\right)} = e^{-\frac{2}{\pi}}$$

26. The shortest distance from the point $(2,0)$ to the curve $y^2 = x^2 + 7$ is

(a) 3

(b) 4

(c) 1

(d) 2

(e) 5

$$d = \sqrt{(x-2)^2 + y^2}$$

$$= \sqrt{2x^2 - 4x + 11}$$

Let $d(x) = \sqrt{2x^2 - 4x + 11}$.

$$d'(x) = \frac{4(x-1)}{2\sqrt{2x^2-4x+11}}$$

x	$-\infty$	1	∞
$d'(x)$	$-$	0	$+$
$d(x)$	$\searrow$	3	$\nearrow$

The shortest distance from the point to the curve is 3.

27. If $f(x) = x^3 + ax^2 + bx$ has two critical numbers at $x = -1$ and $x = 2$, then $a \cdot b =$

(a) 9

(b) $\frac{-3}{2}$

(c) 6

(d) -8

(e) $\frac{-15}{2}$

$f'(-1) = 0 \Rightarrow 3 - 2a + b = 0$

$f'(2) = 0 \Rightarrow 12 + 4a + b = 0$

$9 + 6a = 0$, $a = \frac{-3}{2}$

$b = 2a - 3$, $b = -6$

Thus, $a \cdot b = 9$

28. If $f(x) = \begin{cases} e^x + ax & \text{if} \quad x \le 1 \\ bx^{-1} + x & \text{if} \quad x > 1 \end{cases}$
is differentiable at $x = 1$, then $f'(1) =$

(a) 1

(b) e

(c) 0

(d) 2

(e) 3

If f is continuous at $x=1$ and $\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x) = L$, then f is differentiable at $x=1$ and $f'(1) = L$

$\lim_{x \to 1^+} (bx^{-1} + x) = e + a \Rightarrow b + 1 = e + a$

$f'(x) = \begin{cases} e^x + a, & \text{if } x < 1 \\ -bx^{-2} + 1, & \text{if } x > 1 \end{cases}$

$\lim_{x \to 1^-} (e^x + a) = \lim_{x \to 1^+} (-bx^{-2} + 1) \Rightarrow e + a = -b + 1$

$b = 0$, $a = 1 - e$

Thus, $f'(1) = 1$.

Answer KEY

Q	MM	V1	V2	V3	V4
1	a	b	c	a	d
2	a	c	a	e	c
3	a	a	e	b	a
4	a	b	d	a	b
5	a	a	a	e	b
6	a	e	e	a	e
7	a	e	a	c	b
8	a	a	b	a	b
9	a	c	d	c	e
10	a	c	d	e	c
11	a	c	d	e	a
12	a	b	b	e	c
13	a	b	a	c	d
14	a	c	e	d	e
15	a	a	b	c	a
16	a	e	a	b	b
17	a	a	b	e	b
18	a	a	c	e	d
19	a	b	b	a	a
20	a	a	b	e	a
21	a	c	d	a	e
22	a	c	b	e	b
23	a	a	d	d	d
24	a	d	d	e	b
25	a	b	c	e	d
26	a	a	e	b	b
27	a	b	b	a	b
28	a	c	e	d	a