1. An equation of the tangent line to the curve $y = x + \frac{2}{x}$ at the point $(1, 3)$ is

$y = f'(1)(x-1)+3$

$f'(x) = 1 - \frac{2}{x^2}$, $f'(1) = -1$

$\Rightarrow y = -(x-1)+3$

$= -x+4$

(a) $y = -x + 4$

(b) $y = -2x + 5$

(c) $y = x + 2$

(d) $y = 3x$

(e) $y = 4x - 1$

2. If $y = (2 - x^4)\, e^x$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = -4x^3 e^x + (2-x^4)\cdot e^x$

$= (2-4x^3-x^4)\, e^x$

(a) $(2 - 4x^3 - x^4)\, e^x$

(b) $-4x^3\, e^x$

(c) $(2 - x^4)\, e^x$

(d) $(x^3 - x^4)\, e^x$

(e) $-x\, e^x$

3. If $f(x) = x^3 - \cos x + 2\sin x$, then $f^{(3)}(0) =$

(a) 4

(b) 2

(c) 1

(d) 5

(e) 3

$f'(x) = 3x^2 + \sin x + 2\cos x$

$f''(x) = 6x + \cos x - 2\sin x$

$f^{(3)}(x) = 6 - \sin x - 2\cos x$

$\Rightarrow f^{3}(0) = 6 - 0 - 2 = 4$

4. If $f(x) = \dfrac{\tan^{-1} x}{\cos^{-1} x}$, then $f'(0) =$

(a) $\dfrac{2}{\pi}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{4}{\pi}$

(d) $\dfrac{\pi}{4}$

(e) 0

$$f'(x) = \frac{\frac{1}{x^2+1}\cos^{-1} x - \frac{-1}{\sqrt{1-x^2}}\tan^{-1} x}{(\cos^{-1} x)^2}$$

$$\Rightarrow f'(0) = \frac{\cos^{-1} 0 - \tan^{-1} 0}{(\cos^{-1} 0)^2}$$

$$= \frac{\pi/2 - 0}{(\pi/2)^2} = \frac{2}{\pi}$$

5. $\lim_{x\to 1} \dfrac{\log_2(x^2+1) - \log_2 2}{x-1} =$

$$\lim_{x\to 1} \frac{\log_2(x^2+1) - \log_2 2}{x-1} = \frac{d}{dx}\log_2(x^2+1)\Big|_{x=1}$$
$$= \frac{1}{\ln 2}\frac{2x}{x^2+1}\Big|_{x=1}$$
$$= \frac{1}{\ln 2}$$

(a) $\dfrac{1}{\ln 2}$

(b) $\ln 2$

(c) $\dfrac{2}{\ln 2}$

(d) $-\ln 2$

(e) $-\dfrac{1}{\ln 2}$

6. If $f(3) = 7$ and $f'(3) = \dfrac{1}{2}$, then $(f^{-1})'(7) =$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$
$$\Rightarrow (f^{-1})'(7) = \frac{1}{f'(f^{-1}(7))}$$
$$= \frac{1}{f'(3)}$$
$$= 2$$

(a) 2

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{3}$

(d) 3

(e) $\dfrac{1}{7}$

7. Let $g(x) = [f(\tan x)]^2$. If $f(1) = 1$ and $f'(1) = -1$, then $g'\left(\frac{\pi}{4}\right) =$

(a) -4

(b) -2

(c) 4

(d) 2

(e) 1

$g'(x) = 2\sec^2 x\, f'(\tan x) \cdot f(\tan x)$

$\Rightarrow g'\left(\frac{\pi}{4}\right) = 2\sec^2(\pi/4)\, f'\left(\tan\frac{\pi}{4}\right) \cdot f(\tan \pi/4)$

$= 2 \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} f'(1) \cdot f(1)$

$= -4$

8. If $f(x) = \cos(5x)$, then $f^{(27)}(x) =$

(a) $5^{27} \sin(5x)$

(b) $5^{27} \cos(5x)$

(c) $-5^{27} \sin(5x)$

(d) $-5^{27} \cos(5x)$

(e) $5^{26} \cos(5x)$

$f'(x) = -5\sin(5x)$

$f''(x) = -5^2 \cos(5x)$

$f^{(3)}(x) = 5^3 \sin(5x)$

$f^{(4)}(x) = 5^4 \cos(5x)$

Now, $27 = 4(6) + 3$

$\Rightarrow f^{(27)}(x) = 5^{27} \sin(5x)$

9. If $f(x) = \dfrac{(x-2)\,e^x}{(3-x)^2\sqrt{x+1}}$, then $f'(0) =$

(a) $\dfrac{-4}{27}$

(b) $\dfrac{4}{27}$

(c) $\dfrac{2}{27}$

(d) $\dfrac{-2}{27}$

(e) $\dfrac{-6}{27}$

$$\ln f(x) = \ln(x-2) + x - 2\ln(3-x) - \frac{1}{2}\ln(x+1)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{x-2} + 1 + \frac{2}{3-x} - \frac{1}{2(x+1)}$$

$$\Rightarrow \frac{f'(0)}{f(0)} = -\frac{1}{2} + 1 + \frac{2}{3} - \frac{1}{2} = \frac{2}{3}$$

but, $f(0) = -\frac{2}{9}$

$$\Rightarrow f'(0) = -\frac{2}{9}\left(\frac{2}{3}\right) = -\frac{4}{27}$$

10. The slope of the tangent line to the curve $\cos^2 x + \cos^2 y + \cos(2x+2y) = 0$ at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$ is

(a) -1

(b) $\dfrac{-1}{2}$

(c) 0

(d) 1

(e) $\dfrac{1}{3}$

$$-2\cos x \sin x - 2(\cos y)(\sin y)(y') - (2+2y')\sin(2x+2y) = 0$$

$$\Rightarrow -2\cos^2\frac{\pi}{4} - 2\cos^2\frac{\pi}{4}\, y'(\tfrac{\pi}{4}) - \left(2+2y'(\tfrac{\pi}{4})\right)\underbrace{\sin\pi}_{=0} = 0$$

$$-2\left(\frac{\sqrt{2}}{2}\right)^2 - 2\left(\frac{\sqrt{2}}{2}\right)^2 y'(\tfrac{\pi}{4}) = 0$$

$$y'(\tfrac{\pi}{4}) = -1$$

11. If $y = \sin^{-1}(2x+1)$, then $y' =$

(a) $(-x^2 - x)^{-1/2}$

(b) $(x^2 + x)^{-1/2}$

(c) $-(x^2 + x)^{-1/2}$

(d) $-(-x^2 - x)^{-1/2}$

(e) $(x^2 - x)^{-1/2}$

$$y' = 2\,\frac{1}{\sqrt{1-(2x+1)^2}}$$

$$= \frac{2}{\sqrt{1-4x^2-4x-1}}$$

$$= \frac{2}{2\sqrt{-x^2-x}} = (-x^2-x)^{-1/2}$$

12. An object moves along the x-axis with position at time t $(t \geq 0)$ given by the function

$$s(t) = t^3 - 12t^2 + 36t - 27.$$

The object is speeding up on the time interval

(a) $(2,4) \cup (6,\infty)$

(b) $(0,2) \cup (4,6)$

(c) $(0,4)$

(d) $(4,\infty)$

(e) $(2,6)$

Velocity: $v(t) = 3t^2 - 24t + 36$
$= 3(t^2 - 8t + 12)$
$= 3(t-2)(t-6)$

acceleration: $a(t) = 6t - 24$
$= 6(t-4)$

	0		2		4		6		∞
$v(t)$		+	0	−		−	0	+	
$a(t)$		−		−	0	+		+	
				speeding up				speeding up	

13. If $y = \dfrac{\ln x}{1 + \ln x}$, then $y'' =$

(a) $-\dfrac{3 + \ln x}{x^2(1 + \ln x)^3}$

(b) $-\dfrac{3 + \ln x}{x^2(1 + \ln x)^4}$

(c) $\dfrac{3 + \ln x}{x^2(1 + \ln x)^3}$

(d) $\dfrac{3 + \ln x}{x^2(1 + \ln x)^4}$

(e) $\dfrac{3 + \ln x}{x(1 + \ln x)^3}$

$$y' = \frac{\frac{1}{x}(1+\ln x) - \frac{1}{x}\ln x}{(1+\ln x)^2} = \frac{1}{x(1+\ln x)^2}$$

$$y'' = -\frac{\left[x(1+\ln x)^2\right]'}{x^2(1+\ln x)^4} = -\frac{(1+\ln x)^2 + 2x(1+\ln x)\frac{1}{x}}{x^2(1+\ln x)^4} = -\frac{3+\ln x}{x^2(1+\ln x)^3}$$

14. If $y = (\ln x)^{\cos x}$, then $y' =$

(a) $y\left[\dfrac{\cos x}{x \ln x} - (\sin x)\ln(\ln x)\right]$

(b) $y\left[\dfrac{\cos x}{\ln x} - (\sin x)\ln(\ln x)\right]$

(c) $y\left[\dfrac{\cos x}{x} - (\sin x)\ln(\ln x)\right]$

(d) $y\left[\dfrac{\cos x \ln x}{x} - (\sin x)\ln(\ln x)\right]$

(e) $y\left[\dfrac{\sin x}{x \ln x} - (\cos x)\ln(\ln x)\right]$

$$\ln y = (\cos x)\ln(\ln x)$$

$$\Rightarrow \frac{y'}{y} = -(\sin x)\ln(\ln x) + \frac{\cos x}{x\ln x}$$

$$\Rightarrow y' = y\left[\frac{\cos x}{x\ln x} - (\sin x)\ln(\ln x)\right]$$

15. $\lim\limits_{x\to 0} \dfrac{\sin(2-2\cos x)\tan(1-\cos x)}{(1-\cos^2 x)^2} =$

(a) $\frac{1}{2}$

(b) 2

(c) $\frac{1}{4}$

(d) 4

(e) 0

$$f(x) = \frac{\sin(2-2\cos x)\tan(1-\cos x)}{(1-\cos^2 x)^2} = 2\,\frac{\sin 2(1-\cos x)}{2(1-\cos x)}\,\frac{\tan(1-\cos x)}{1-\cos x}\,\frac{1}{(1+\cos x)^2}$$

But, $\lim\limits_{x\to 0} \dfrac{\sin 2(1-\cos x)}{2(1-\cos x)} = 1$

and $\lim\limits_{x\to 0} \dfrac{\tan(1-\cos x)}{1-\cos x} = 1$

$$\Rightarrow \lim_{x\to 0} f(x) = 2\times 1\times 1\times \frac{1}{4} = \frac{1}{2}$$

16. The graph of $f(x) = \dfrac{\sec x}{1+\tan x}$, $0 \le x \le \pi$, has a horizontal tangent at the point

(a) $x = \frac{\pi}{4}$

(b) $x = \frac{\pi}{3}$

(c) $x = \frac{\pi}{2}$

(d) $x = \frac{3\pi}{4}$

(e) $x = \frac{\pi}{6}$

$$f'(x) = \frac{\sec x\tan x\,(1+\tan x) - \sec^2 x\,\sec x}{(1+\tan x)^2}$$

$$= \frac{\sec x\,(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x\,(\tan x - 1)}{(1+\tan x)^2}, \text{ since } 1+\tan^2 x = \sec^2 x$$

$$f'(x) = 0 \Leftrightarrow \tan x = 1, \quad x = \frac{\pi}{4}$$

17. Equations of the tangent lines to the curve $y = \dfrac{1-x}{1+x}$ that are parallel to the line $x + 2y = 2$ are

(a) $y = \dfrac{1-x}{2}$ and $y = \dfrac{-7-x}{2}$

(b) $y = \dfrac{x-1}{2}$ and $y = \dfrac{x+7}{2}$

(c) $y = -x - \dfrac{1}{2}$ and $y = -x + \dfrac{7}{2}$

(d) $y = \dfrac{-x-1}{2}$ and $y = \dfrac{7-x}{2}$

(e) $y = \dfrac{2-x}{2}$ and $y = \dfrac{-6-x}{2}$

$$y' = \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}.$$

We require $y' = -\frac{1}{2}$, that is,

$$\frac{-2}{(1+x)^2} = -\frac{1}{2} \Leftrightarrow (1+x)^2 = 4$$

$$1+x = 2 \Rightarrow x = 1$$

$$x+1 = -2 \Rightarrow x = -3$$

$$\Rightarrow y = -\frac{1}{2}(x-1) + \underbrace{y(1)}_{=0} = \frac{1-x}{2}$$

and $y = -\frac{1}{2}(x+3) + \underbrace{y(-3)}_{=-2}$

$$= -\frac{1}{2}(x+3) - 2 = \frac{-x-7}{2}$$

18. A particle is moving along the curve $y = \sqrt{-x}$. As it reaches the point $(-4, 2)$, the y-coordinate is decreasing at a rate of $2\,cm/s$. Then the x-coordinate of the point at that instant

(a) is increasing at a rate of $8\,cm/s$

(b) is decreasing at a rate of $8\,cm/s$

(c) is increasing at a rate of $4\,cm/s$

(d) is decreasing at a rate of $4\,cm/s$

(e) does not change

$$\left.\frac{dy}{dt}\right|_{(-4,2)} = -2 \text{ cm/s}$$

$$\frac{dy}{dt} = -\frac{1}{2\sqrt{-x}}\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -2\sqrt{-x}\,\frac{dy}{dt}$$

$$\left.\frac{dx}{dt}\right|_{(-4,2)} = -2\,(2)\,(-2)$$

$$= 8 \text{ cm/s}$$

19. Let f be a differentiable function. If $f(6) = -3$ and

$$\frac{d}{dx}[xf(3x)] = x^2 - 1,$$

then $f'(6) =$

$f(3x) + 3x\, f'(3x) = x^2 - 1$

Let $x = 2$.

$\Rightarrow \underbrace{f(6)}_{=-3} + 6 f'(6) = 3$

$\Rightarrow f'(6) = \frac{6}{6} = 1$

(a) 1

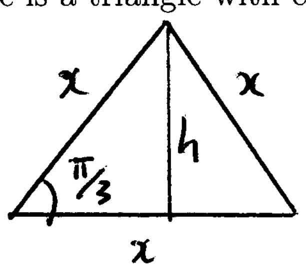
(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{6}$

(e) $-\frac{2}{3}$

20. The sides of an equilateral triangle are increasing at a rate of $1\,cm/min$. At what rate is the area of the triangle increasing when the sides are $4\,cm$ long?
(**Hint**: An equilateral triangle is a triangle with equal sides)



$A = \frac{hx}{2}$

$\sin\frac{\pi}{3} = \frac{h}{x} \Rightarrow h = \frac{\sqrt{3}}{2}x$

$\Rightarrow A = \frac{\sqrt{3}}{4}x^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\, 2x \frac{dx}{dt}$

$\Rightarrow \left.\frac{dA}{dt}\right|_{x=1} = \frac{\sqrt{3}}{4} \times 2 \times 4 \times 1$

$= 2\sqrt{3}\ cm^2/min$

(a) $2\sqrt{3}\ cm^2/min$

(b) $3\sqrt{3}\ cm^2/min$

(c) $4\sqrt{3}\ cm^2/min$

(d) $5\sqrt{3}\ cm^2/min$

(e) $\sqrt{3}\ cm^2/min$

Answer KEY

Q	MM	V1	V2	V3	V4
1	a	e	d	b	c
2	a	d	d	e	c
3	a	e	b	b	a
4	a	c	b	b	a
5	a	a	a	b	d
6	a	e	d	c	b
7	a	c	e	a	a
8	a	e	b	c	a
9	a	b	b	d	b
10	a	b	c	d	c
11	a	e	d	e	a
12	a	b	e	c	a
13	a	e	e	c	d
14	a	b	e	d	c
15	a	e	e	c	d
16	a	c	a	b	b
17	a	d	b	e	e
18	a	d	e	c	d
19	a	b	d	b	d
20	a	a	c	e	e