1. The slope of the tangent line to the curve $y = e^x + x$ at the point $(0, 1)$ is given by

(a) $\lim\limits_{x \to 0} \dfrac{e^x + x - 1}{x}$

(b) $\lim\limits_{x \to 0} \dfrac{e^x + x}{x - 1}$

(c) $\lim\limits_{x \to 0} \dfrac{e^x + x - 1}{x - 1}$

(d) $\lim\limits_{x \to 0} \dfrac{e^x + x + 1}{x}$

(e) $\lim\limits_{x \to 0} \dfrac{e^x + x}{x}$

$m = \lim\limits_{x \to 0} \dfrac{y-1}{x}$

$= \lim\limits_{x \to 0} \dfrac{e^x + x - 1}{x}$

2. Which one of the following statements is **TRUE**?

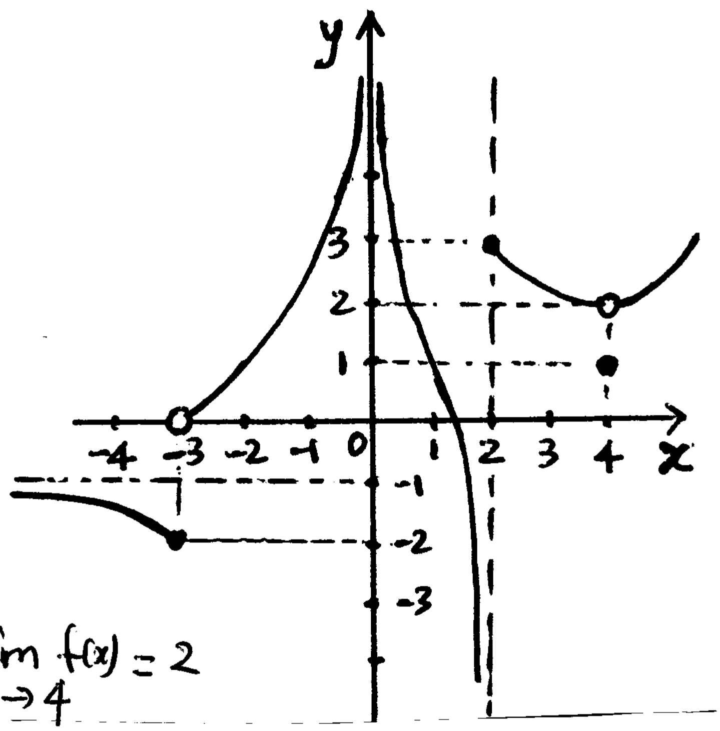
(a) $\lim\limits_{x \to 0} f(x) = \infty$

(b) $\lim\limits_{x \to -3} f(x) = -2$

(c) $\lim\limits_{x \to 1} f(x) = 0$

(d) $\lim\limits_{x \to 4} f(x) = 1$

(e) $\lim\limits_{x \to -\infty} f(x) = \infty$



$\lim\limits_{x \to -3} f(x)$ DNE ; $\lim\limits_{x \to 4} f(x) = 2$

$\lim\limits_{x \to 1} f(x) = 1$; $\lim\limits_{x \to -\infty} f(x) = -1$

3. $\lim\limits_{x\to 1} \dfrac{\sqrt{x+8}-3}{x-1} =$

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{9}$

(d) 3

(e) 1

$$\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1}$$
$$= \lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} \cdot \frac{(\sqrt{x+8}+3)}{\sqrt{x+8}+3}$$
$$= \lim_{x\to 1} \frac{x+8-9}{(x-1)(\sqrt{x+8}+3)}$$
$$= \lim_{x\to 1} \frac{1}{\sqrt{x+8}+3}$$
$$= \frac{1}{6}$$

4. If $[\![x]\!]$ is the greatest integer less than or equal to x, then $\lim\limits_{x\to 1} [\![\frac{x}{2} - 1]\!] =$

(a) -1

(b) 0

(c) 1

(d) -2

(e) does not exist

$$\lim_{x\to 1} \left(\frac{x}{2}-1\right) = -\frac{1}{2}$$
$$\Rightarrow \lim_{x\to 1} [\![\frac{x}{2}-1]\!] = -1$$

5. Let

$$f(x) = \begin{cases} 3 - \dfrac{1}{2}x & \text{if} \quad x < 2 \\ \sqrt{x+c} & \text{if} \quad x \geq 2. \end{cases}$$

If $\lim\limits_{x \to 2} f(x)$ exists, then $c^3 + 3c - 7 =$

(a) 7

(b) -3

(c) 0

(d) -5

(e) 10

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(3 - \frac{x}{2}\right) = 2$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \sqrt{x+c} = \sqrt{2+c}$$

$$\Rightarrow \quad 2 = \sqrt{2+c} \Rightarrow c = 2$$

Thus, $c^3 + 3c - 7 = 7$

6. $\lim\limits_{x \to 0} x^2 \cos \dfrac{1}{x} =$

(a) 0

(b) 1

(c) 2

(d) does not exist

(e) -1

$-1 \leq \cos \frac{1}{x} \leq 1$, for any $x \neq 0$

$$\Rightarrow -x^2 \leq x^2 \cos \frac{1}{x} \leq x^2$$

$\lim\limits_{x \to 0} x^2 = 0$ and $\lim\limits_{x \to 0} (-x^2) = 0$

$$\Rightarrow \lim_{x \to 0} x^2 \cos \frac{1}{x} = 0$$

by squeeze theorem

7. $\lim_{x\to\infty}(5 + x - 6x^5) =$

(a) $-\infty$

(b) 0

(c) -6

(d) 1

(e) ∞

$$\lim_{x\to\infty}(5+x-6x^5) = \lim_{x\to\infty} x^5\left(\frac{5}{x^5}+\frac{1}{x^4}-6\right)$$

$$= -\infty,$$

since $\lim_{x\to\infty}\frac{1}{x^4} = 0$

and $\lim_{x\to\infty}\frac{5}{x^5} = 0$

8. Let $f(x) = 4x - 3$. A possible value of δ such that if $0 < |x - 2| < \delta$, then $|f(x) - 5| < 1$ is

(a) 0.23

(b) 0.50

(c) 0.33

(d) 0.75

(e) 0.27

$|f(x)-5| < 1$

$\Leftrightarrow |4x-3-5| < 1$

$\Leftrightarrow |4(x-2)| < 1$

$\Leftrightarrow |x-2| < \frac{1}{4}$

Now, let any number δ such that $0 < \delta < \frac{1}{4} = 0.25$.

If $|x-2| < \delta$, then $|f(x)-5| < 1$.

Thus, $\delta = 0.23$ is a possible choice of δ.

9. The function $f(x) = \dfrac{1 - \ln x}{e - x}$ is continuous on

(a) $(0, e) \cup (e, \infty)$

(b) $(0, \infty)$

(c) $(0, 4)$

(d) $(1, 2e)$

(e) $(-\infty, e) \cup (e, \infty)$

f is continuous on its domain of definition.

$D_f = \{x \in \mathbb{R} / \ x > 0 \text{ and } x \neq e\}$

$= (0, e) \cup (e, \infty)$

10. Using the Intermediate Value Theorem, the equation $x^3 - x = 2 - \sqrt{x}$ has a root in the interval

(a) $(1, 2)$

(b) $(0, 1)$

(c) $(2, 3)$

(d) $(3, 4)$

(e) $(2, 4)$

Let $f(x) = x^3 - x + \sqrt{x} - 2$.

f is continuous on $[0, \infty)$

$f(0) = -2$; $f(1) = -1$

$f(2) = 4 + \sqrt{2}$

$f(3) = 22 + \sqrt{3}$

$f(4) = 60$.

The equation $f(x) = 0$ has a root in the interval $(1, 2)$ by the intermediate value theorem

11. $\lim_{x \to -\infty} \tan^{-1}\left(\frac{\sqrt{1+4x^6}}{1-2x^3}\right) =$

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{-\pi}{4}$

(d) $\frac{-\pi}{2}$

(e) $\frac{\pi}{3}$

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{1-2x^3} = \lim_{x\to-\infty} \frac{|x|^3\sqrt{\frac{1}{x^6}+4}}{x^3\left(\frac{1}{x^3}-2\right)}$$

$$= \lim_{x\to-\infty} \frac{-x^3\sqrt{\frac{1}{x^6}+4}}{x^3\left(\frac{1}{x^3}-2\right)}$$

$$= \lim_{x\to-\infty} -\frac{\sqrt{\frac{1}{x^6}+4}}{\frac{1}{x^3}-2}$$

$$= 1.$$

But, $\tan^{-1}(1) = \pi/4$

$$\Rightarrow \lim_{x\to-\infty} \tan^{-1}\left(\frac{\sqrt{1+4x^6}}{1-2x^3}\right) = \pi/4$$

12. Let $f(x) = \frac{x^2-x-2}{x^2-3x+2}$.

Which one of the following statements is **TRUE**?

(a) f has a removable discontinuity at $x = 2$ and an infinite discontinuity at $x = 1$

(b) f has two removable discontinuities at $x = 1$ and $x = 2$

(c) f has two infinite discontinuities at $x = 1$ and $x = 2$

(d) f has a removable discontinuity at $x = 1$ and an infinite discontinuity at $x = 2$

(e) f has a jump discontinity at $x = 2$ and a removable discontinuity at $x = 1$

$$f(x) = \frac{(x-2)(x+1)}{(x-2)(x-1)} \; ; \quad \lim_{x\to2} f(x) = \lim_{x\to2} \frac{x+1}{x-1} = 3$$

and $\lim_{x\to1^+} f(x) = \lim_{x\to1^+} \frac{x+1}{x-1} = \infty$.

Thus, $x=2$ is a removable discontinuity
$x=1$ is an infinite discontinuity

13. All the horizontal asymptote(s) of the curve $y = \dfrac{9e^x + 2}{3e^x - 4}$ is (are)

(a) $y = 3 \text{ and } y = -\dfrac{1}{2}$

(b) $y = 3 \text{ and } y = -\dfrac{1}{4}$

(c) $y = 3 \text{ only}$

(d) $y = 9 \text{ and } y = \dfrac{1}{4}$

(e) $y = -3 \text{ and } y = 2$

$$\lim_{x\to\infty} \frac{9e^x+2}{3e^x-4} = \lim_{x\to\infty} \frac{e^x(9+2e^{-x})}{e^x(3-4e^{-x})}$$

$$= \lim_{x\to\infty} \frac{9+2e^{-x}}{3-4e^{-x}}$$

$$= \frac{9}{3} = 3.$$

$$\lim_{x\to-\infty} \frac{9e^x+2}{3e^x-4} = \frac{2}{-4} = -\frac{1}{2}$$

Thus, $y=3$ and $y=-\frac{1}{2}$ are the horizontal asymptotes.

14. If the tangent line to the curve $y = f(x)$ at $(4,3)$ passes through the point $(0,2)$, then $f'(4) =$

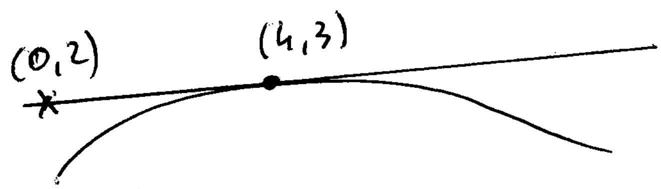
(a) $\dfrac{1}{4}$

(b) 4

(c) $\dfrac{-1}{4}$

(d) -4

(e) 0



$$f'(4) = \frac{3-2}{4-0} = \frac{1}{4}$$

($f'(4)$ is the slope of this tangent line)

15. Suppose you know that the derivative of $\tan x$ is $\sec^2 x$ for all x in its domain. Then

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{4x - \pi} =$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{x - \pi/4}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\tan x - \tan(\frac{\pi}{4})}{x - \pi/4} = \sec^2(\pi/4)$$

$$= 2$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{4(x-\pi)} = \frac{2}{4} = \frac{1}{2}$$

(a) $\frac{1}{2}$

(b) 2

(c) 0

(d) 1

(e) -2

16. If $f(x) = \begin{cases} 3x^2 & \text{if} \quad x < 1 \\ ax^2 - bx + 3 & \text{if} \quad 1 \le x \le 3 \\ 2x - a + b & \text{if} \quad x > 3 \end{cases}$

is continuous on $(-\infty, \infty)$, then $f(2) =$

f is continuous at $x=1$ and $x=3$.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow 3 = a - b + 3$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) \Rightarrow 9a - 3b + 3 = 6 - a + b$$

$$\begin{cases} a - b = 0 \\ 10a - 4b = 3 \end{cases} \Rightarrow a = b = \frac{1}{2}.$$

Thus, $f(2) = \frac{1}{2}(2)^2 - \frac{1}{2}(2) + 3 = 4$

(a) 4

(b) -3

(c) 8

(d) 0

(e) -5

17. If $f(x) = x|x-1|$, then $f'_-(1) =$
(The left-hand derivative of f at $x = 1$)

(a) -1

(b) 1

(c) 0

(d) 2

(e) does not exist

$$f'_-(1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$$
$$= \lim_{x\to 1^-} \frac{x|x-1|}{x-1}$$
$$= \lim_{x\to 1^-} \frac{-x(x-1)}{x-1}$$
$$= \lim_{x\to 1^-} -x$$
$$= -1$$

18. Let $f(x) = \dfrac{x^3+x}{3x-2x^2}$.
Which one of the following statements is **TRUE**?

(a) $\displaystyle\lim_{x\to\frac{3}{2}^+} f(x) = -\infty$

(b) $\displaystyle\lim_{x\to 0} f(x)$ does not exist

(c) $\displaystyle\lim_{x\to\frac{3}{2}} f(x) = \infty$

(d) $f(x)$ has vertical asymptotes at $x = 0$ and $x = \dfrac{3}{2}$

(e) $f(x)$ has no vertical asymptotes

$$f(x) = \frac{x(x^2+1)}{x(3-2x)}$$

$$\lim_{x\to\frac{3}{2}^+} f(x) = \lim_{x\to\frac{3}{2}^+} \frac{x^2+1}{3-2x} = -\infty$$

f has a vertical asymptote at $x = \frac{3}{2}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{x^2+1}{3-2x} = \frac{1}{3}$$

$$\left\{\begin{array}{l} \displaystyle\lim_{x\to\frac{3}{2}^-} f(x) = \infty \\ \displaystyle\lim_{x\to\frac{3}{2}^+} f(x) = -\infty \end{array}\right. \Longrightarrow \lim_{x\to\frac{3}{2}} f(x) \neq \infty$$

19. $\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)=$

(a) -1

(b) 0

(c) ∞

(d) 2

(e) $-\infty$

$$\frac{1}{x-1}+\frac{1}{x^2-3x+2}=\frac{1}{x-1}+\frac{1}{(x-1)(x-2)}$$

$$=\frac{x-1}{(x-1)(x-2)}$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x-2}\right)=\lim_{x\to 1}\frac{x-1}{(x-1)(x-2)}$$

$$=\lim_{x\to 1}\frac{1}{x-2}$$

$$=-1$$

20. Let $f(x)=\frac{x}{1+x}$. Then $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=$

(a) $\frac{1}{(1+x)^2}$

(b) $\frac{-1}{(1+x)^2}$

(c) $\frac{x}{(1+x)^2}$

(d) $\frac{-x}{(1+x)^2}$

(e) $\frac{x^2}{(1+x)^2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x+h}{1+x+h}-\frac{x}{1+x}}{h}$$

$$=\lim_{h\to 0}\frac{(1+x)(x+h)-x(1+x+h)}{h(1+x+h)(1+x)}$$

$$=\lim_{h\to 0}\frac{h}{h(1+x+h)(1+x)}$$

$$=\lim_{h\to 0}\frac{1}{(1+x+h)(1+x)}$$

$$=\frac{1}{(1+x)^2}$$

Answer KEY

Q	MM	V1	V2	V3	V4
1	a	e	d	a	b
2	a	a	c	a	c
3	a	c	c	d	e
4	a	a	d	d	d
5	a	b	d	d	e
6	a	c	e	b	c
7	a	e	b	b	d
8	a	c	d	b	b
9	a	e	c	d	b
10	a	d	a	b	b
11	a	d	c	d	d
12	a	e	b	e	d
13	a	a	e	b	d
14	a	e	a	c	a
15	a	b	c	c	b
16	a	b	c	a	e
17	a	a	a	a	a
18	a	c	d	e	e
19	a	a	a	e	c
20	a	e	d	a	d