King Fahd University of Petroleum & Minerals

Final Examination

Faculty: Science	**Department: Mathematics**
Semester: 172	**Course Name: Actuarial Risk & Credibility Theory**
Instructor: Abedalhay Elmughrabi	**Course No: AS 483**
Exam Date: May 7th, 2018	**Exam Time: 08:00 AM – 11:00 AM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks
1	8 points		11	8 points	
2	9 points		12	8 points	
3	8 points		13	8 points	
4	9 points		14	8 points	
5	8 points		15	8 points	
6	8 points		16	8 points	
7	9 points		17	8 points	
8	8 points				
9	8 points				
10	9 points				

Obtained Total:

/ 140

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **17** questions.
3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
4. Calculators cannot be exchanged during the examination.
5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
6. Cell phones must be turned off and placed under your bench facedown.
7. No questions are allowed during the exam.
8. All material related to the course should be put away
9. Final correct answers have significant weights
10. Answers without calculations/steps will receive zero marks.
11. Be clean, neat and tidy, else your work may not be marked
12. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

Question 1 (8 Points):

Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with rate θ and let θ be gamma (α,β) with density

$$g(\theta) = \frac{\beta^{\alpha}}{(\alpha-1)!}\,\theta^{\alpha-1}\,e^{-\beta\theta} \qquad \theta > 0$$

Find the Bayes estimator of θ under the square error loss?

Question 2 (9 Points):

An insurer has excess-of-loss reinsurance on auto insurance. You are given:
(i) Total expected losses in the year 2001 are 10,000,000.
(ii) In the year 2001 individual losses have a Pareto distribution with

$$F(x) = 1 - \left(\frac{2000}{x + 2000}\right)^2, x > 0$$

(iii) Reinsurance will pay the excess of each loss over 3000.
(iv) Each year, the reinsurer is paid a ceded premium, Cyear; equal to 110% of the expected losses covered by the reinsurance.
(v) Individual losses increase 5% each year due to inflation.
(vi) The frequency distribution does not change.
(a) Calculate C_{2001}
(b) Calculate $\frac{C_{2002}}{C_{2001}}$

Question 3 (8 Points):

The amount of money in dollars that Clark received in 2010 from his investment in futures follows a Pareto distribution with parameters $\alpha = 3$ and θ. Annual inflation in the US from 2010 to 2011 is i%. The 80^{th} percentile of the earning size in 2010 equals the mean earning size in 2011. If Clark's investment income keeps up with inflation but is otherwise unaffected, determine i?

Questions 4 (9 Points):

You are given:

(i) Conditional on $Q = q$, the random variables X_1 , X_2 , …, X_m are independent and follow a Bernoulli distribution with parameter q.

(ii) $S_m = X_1 + X_2 + \ldots + X_m$
(iii) The distribution of Q is beta with $a = 1$, $b = 99$, and $\theta = 1$.

Determine the variance of the marginal distribution of S_{101}.

Questions 5 (8 Points):

An insurance company sells two types of policies with the following characteristics:

Type of Policy	Proportion of Total Policies	Poisson Annual Claim Frequency
I	θ	$\lambda = 0.50$
II	$1-\theta$	$\lambda = 1.50$

A randomly selected policyholder is observed to have one claim in Year 1.

For the same policyholder, determine the Bühlmann credibility factor Z for Year 2.

Question 6 (8 Points):

You are given:

(i)

Claim Size	Number of Claims
(0, 50]	30
(50, 100]	36
(100, 200]	18
(200, 400]	16

(ii) Claim sizes within each interval are uniformly distributed.

(iii) The second moment of the uniform distribution on (a b,] is $\frac{b^3-a^3}{3(ab-a)}$.

Estimate $E[(X \wedge 350)^2]$, the second moment of the claim size distribution subject to a limit of 350.

Question 7 (9 Points):

Annual aggregate losses for a dental policy follow the compound Poisson distribution with $\lambda = 3$. The distribution of individual losses is:

Loss	Probability
1	0.4
2	0.3
3	0.2
4	0.1

Calculate the probability that aggregate losses in one year do not exceed 3.

Questions 8 (8 Points):

The loss severity random variable X follows the exponential distribution with mean 10,000. Determine the coefficient of variation of the excess loss variable Y = max(X – 30000, 0).

Questions 9 (8 Points):

For a warranty product you are given:

(i) Paid losses follow the lognormal distribution with 13.294 μ = and σ = 0.494 .

(ii) The ratio of estimated unpaid losses to paid losses, y, is modeled by

$$y = 0.801\, x^{0.851}\, e^{-0.747x}$$

where x = 2006 − contract purchase year.

The inversion method is used to simulate four paid losses with the following four uniform (0,1) random numbers: 0.2877 0.1210 0.8238 0.6179. Using the simulated values, calculate the empirical estimate of the average unpaid losses for purchase year 2005.

Questions 10 (9 Points):

An insurance company is revising rates based on old data. The expected number of claims for full credibility is selected so that the observed total claims will be within 5% of the true value 90% of the time. Individual claim amounts have pdf 1/200,000, $0<x<200{,}000$, and the number of claims has the poison distribution. The recent experience consists of 1,082 claims. Determine the credibility, Z, to be assigned to the recent experience. Use the normal approximation.

Questions 11 (8 Points):

You are given:

(i) The prior distribution of the parameter **θ** has probability density function:

$$\pi(\theta) = \frac{1}{\theta^2} \qquad 1 < \theta < \infty$$

(ii) Given **θ** $=\theta$, claim sizes follow a Pareto distribution with parameters α = 2 and θ .

A claim of 3 is observed. Calculate the posterior probability that **θ** exceeds 2.

Questions 12 (8 Points):

You are given four classes of insureds, each of whom may have zero or one claim, with the following probabilities:

Class	Number of Claims	
	0	1
I	0.9	0.1
II	0.8	0.2
III	0.5	0.5
IV	0.1	0.9

A class is selected at random (with probability ¼), and four insureds are selected at random from the class. The total number of claims is two. If five insureds are selected at random from the same class, estimate the total number of claims using Bühlmann-Straub credibility.

Question 13 (8 Points):

For a portfolio of independent risks, the number of claims for each risk in a year follows a Poisson distribution with means given in the following table:

Class	Mean Number of Claims per Risk	Number of Risks
1	1	900
2	10	90
3	20	10

You observe x claims in Year 1 for a randomly selected risk. The Bühlmann credibility estimate of the number of claims for the same risk in Year 2 is 11.983. Determine x.

Questions 14 (8 Points):

A dental plan is designed so that a deductible of 100 is applied to annual dental charges. The reimbursement to the insured is 80% of the remaining dental charges subject to an annual maximum reimbursement of 1000.

You are given

i. The annual dental charges for each insured are exponentially distributed with mean 100.
ii. Use the following Uniform (0,1) random numbers and the inversion method to generate four values of annual dental charges.

0.3 0.92 0.7 0.08

Calculate the average annual reimbursement for this simulation?

Question 15 (8 Points):

A company has determined that the limited fluctuation full credibility standard is 2000 claims if:

(i) The total number of claims is to be within 3% of the true value with probability p.

(ii) The number of claims follows a Poisson distribution.

The standard is changed so that the total cost of claims is to be within 5% of the true value with probability p, where claim severity has probability density function:

$$f(x) = \frac{1}{10{,}000}, \quad 0 < x < 10{,}000$$

Using limited fluctuation credibility, determine the expected number of claims necessary to obtain full credibility under the new standard.

Questions 16 (8 Points):

Losses for a warranty product follow the lognormal distribution with underlying normal mean and standard deviation of 5.6 and 0.75 respectively.

You use simulation to estimate claim payments for a number of contracts with different deductibles.

The following are four uniform (0,1) random numbers:

0.6217 0.9941 0.8686 0.0485

Using these numbers and the inversion method, calculate the average payment per loss for a contract with a deductible of 100.

Question 17 (8 Points):

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - 0.3e^{-0.00001x}, & x \geq 0 \end{cases}$$

For the distribution above, determine the following:

- The loss elimination ratio with an ordinary deductible of 5,000.
- The effect of inflation at 10% on an ordinary deductible of 5,000 applied to the distribution.