KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS

STAT 319: Probability & Statistics for Engineers & Scientist
Semester 171, First Major Exam
Monday October 23, 2017

Please circle your instructor's name:

E. Al-Sawi M. Malik M. Omar M. Saleh

Name: ____ ID #:___________________________

Important Note:

- Formula sheet will be provided to you in exam. You are not allowed to bring, with you, formula sheet or any other printed/written paper.
- Mobiles are not allowed in exam. If you have your mobile with you, turn it off and put it under your seat so that it is visible to proctor.
- Show all your work including formulas, intermediate steps and final answer. No points for answer without justification.
- Round up to 4 decimal points if needed.
- Make sure you have 6 unique pages of exam paper (including this title page).

Question No	Full Marks	Marks Obtained
1	10	
2	5	
3	24	
4	12	
5	5	
6	4	
Total	60	

Q1:

a) The number of cars entering a parking lot is a random variable having a Poisson distribution with a mean of 4 per hour. The lot holds only 12 cars. Find the probability that the lot will fill up in the two hour. (Assume that all cars stay in the lot longer than two hour).

b) The time to failure (in hours) of a bearing in a motorized shaft is modeled as a Weibull random variable with $\beta = 1/3$ and $\delta = 2000$ hours. Find the probability that a bearing will last at most 10,000 hours?

c) Suppose that A, B & C are independent events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{6}$ & $P(C) = \frac{1}{2}$. Find the probability that at exactly one of the events will occur.

d) You purchase a certain product. The manual states that the lifetime T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = e^{-\frac{t}{5}}, t > 0$$

I purchase the product and use it for two years without any problems. What is the probability that it breaks down in the third year?

Q2: An oil exploration firm is to drill ten wells, with each having probability 0.1 of successfully producing oil. It costs the firm $10000 to drill each well. A successful well will bring in oil worth $500000. Find the firm's expected gain from the ten wells.

Q3: A salesman has a scheduled two appointments to sell items. He feels that his first appointment will lead to a sale with probability 0.3. He also feels that the second will lead to a sale with probability 0.6, and that results from the two appointments are independent. What is the probability mass function of the number of sales that he made?

$P(X = x)$	0.28	0.54	0.18

Q4: Plastic bottles are inspected for flaws before shipping. Suppose the proportion of bottles that actually have a flaw is 0.0002. If a bottle has a flaw, the probability is 0.995 that it will fail the inspection. If a bottle does not have a flaw, the probability is 0.99 that it will pass the inspection.

a. What percentage of the bottles fail inspection overall?

b. If a bottle fails the inspection, what is the probability that it has a flow?

c. Which of the following is more interpretation of the answer to part b above? Check the best answer)
   i. Most bottles that fail inspection do not have a flaw.
   ii. Most bottles that pass inspection do have a flow.
   iii. Both 1 and 2 have equal probability.

Q5: The probability density function of the time X (in minutes) that a flight from Dammam to Jeddah arrives earlier or later than its scheduled arrival is given by

$$f(x) = \begin{cases} c(36 - x^2) & -6 \leq x \leq 6 \\ 0 & elsewhere \end{cases}$$

Where the negative values of x indicate flight arriving early, while positive values of x indicate flight arriving late.

a. Find the constant c so that the function is a valid probability density function.

a. If a flight is more than 3 minutes late, find the probability it will be less than 5 minutes late.

b. Find the probability that one of these flights will arrive between 1 and 3 minutes earlier than the scheduled arrival.

c. Find the expected value of X

Q6: It is known that semiconductors produced by a certain company will be defective with probability 0.01, independent of each other. Semiconductor are sold in packs of size 10. A money-back guarantee is offered if a pack contains more 1 defective semiconductors.

a. What is the probability of sales results in the customers getting their money back?

b. If someone buys 3 packages, what is the probability that he will return exactly 1 of them?

Formula Sheet

Descriptive Statistics

- $\bar{x} = \frac{\sum x}{n}$ or $\bar{x} = \frac{\sum xf}{\sum f}$
- $s = \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}}$ or $s = \sqrt{\frac{\sum x^2 f - n\bar{x}^2}{n-1}}$
- $R_\alpha = \frac{\alpha(n+1)}{100}$ & $P_\alpha = X_{(i)} + d\left(X_{(i+1)} - X_{(i)}\right)$

Probability

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$
- $P\left(A_j \middle| B\right) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P\left(B \middle| A_j\right)}{\sum_{i=1}^{k} P(A_i)P(B|A_i)}, \quad j = 1, 2, \ldots, k$

Random Variables

- $\mu = E(X) = \sum xP(X = x)$ or $\mu = E(X) = \int xf(x)dx$
- $\sigma^2 = E(x - \mu)^2 = E(x)^2 - \left(E(X)\right)^2$
- $P(X = x) = C_x^n p^x (1-p)^{n-x}, \ x = 0, 1, 2, \ldots, n, \ \mu = np$ & $\sigma = \sqrt{np(1-p)}$
- $P(X = x) = \frac{C_x^K C_{n-x}^{N-K}}{C_n^N}, \ x = max\{0, n + K - N\}$ to $min\{K, n\}$,

  $\mu = n\frac{K}{N}$ & $\sigma = \sqrt{n\frac{K}{N}\left(1 - \frac{K}{N}\right)}\sqrt{\frac{N-n}{N-1}}$
- $P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, x = 0, 1, 2, \ldots; \quad \mu = \lambda t$ & $\sigma = \sqrt{\lambda t}$
- $f(x) = \frac{1}{b-a}, \quad a \le x \le b; \quad \mu = \frac{b+a}{2}$ & $\sigma = \sqrt{\frac{(b-a)^2}{12}}$
- $f(x) = \lambda e^{-\lambda x}, x > 0;$ $where$ $F(x) = 1 - e^{-\lambda x}$ and $\mu = \frac{1}{\lambda}$ & $\sigma = \frac{1}{\lambda}$
- $f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^\beta}, \ x > 0;$ $where$ $F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^\beta}$

  $\mu = \delta\Gamma\left(1 + \frac{1}{\beta}\right)$ & $\sigma^2 = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \mu^2$