KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS

Term 171

STAT 302 Exam #1

Name: ______________________________ ID #: ______________

- Show all details.
- If there is a rule you are using, write down that rule.
- Always give reasons
- Answers without justification are not accepted
- Do not make statements without proof.

Some Useful Formula

$$\sum_{1}^{n} i = \frac{n(n+1)}{2}, \sum_{1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

$$\textit{The Gamma Density } f_Y(y) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} y^{\alpha-1} e^{-\frac{y}{\beta}},$$

$$\textit{where } \Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx, E(Y) = \alpha\beta, V(Y) = \alpha\beta^2, M_Y(t) = (1-\beta t)^{-\alpha}$$

Question	Maximum Marks	Marks Obtained
1	4	
2	6	
3	6	
4	4	
5	7	
6	6	
7	7	
Bonus	10	
Total	40	

1) Let Y_1, Y_2, Y_3 be a random sample from a Bernoulli distribution with parameter p. Is the statistic $Y_1Y_2 + Y_3$ sufficient for p? Explain. *(3 marks)*

2) Let $Y_1, \cdots, Y_n$ be a random sample from $f_Y(y|\theta) = \begin{cases} \frac{1}{\theta} e^{-y/\theta}, & y > 0 \\ 0 & otherwise \end{cases} \;\; ; \;\; \theta > 0$

Find an unbiased estimator for $\frac{1}{\theta}$. *(5 marks)*

3) Let $Y_1, \cdots, Y_n$ be a random sample from $f_Y(y|\theta) = \begin{cases} e^{-(y-\theta)}, & y > 0 \\ 0 & otherwise \end{cases}$

a) Is $Y_{(1)}$ a sufficient statistic for θ? Explain *(2 marks)*

b) Is $Y_{(1)}$ an unbiased estimator for θ? Explain. *(2 marks)*

c) Find $MSE\left(Y_{(1)}\right)$. *(2 marks)*

4) Let $Y_1, \cdots, Y_n$ be a random sample from a distribution with $E(Y) = \mu$, and finite variance σ^2. Let $\widehat{\theta_1} = \bar{Y}$, and $\widehat{\theta_2} = \frac{2}{n(n+1)} \sum_1^n iY_i$.

a) Show that $\widehat{\theta_2}$ is consistent for estimating μ. *(2 marks)*

b) Find the relative efficiency of $\widehat{\theta_1}$ to $\widehat{\theta_2}$. *(2 marks)*

5) Suppose that $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ are independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively.

a) Show that the random variable

$$U_n = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2 + \sigma_2^2)/n}}$$

satisfies the conditions of the central limit theorem, and thus its conclusion.

(3 marks)

b) Assuming part (a) above, consider the following experiment which is designed to test whether operator A or operator B gets the job of operating a new machine. Each operator is timed on 50 independent trials involving performance of a certain task. If the sample means for the 50 trials differ by more than 1 second, the operator with the smaller mean time gets the job. If the standard deviations of times for both operators are assumed to be 2 seconds, what is the probability that operator A will get the job even though both operators have equal ability? *(4 marks)*

6) In a study of consumer satisfaction involving 1030 customers, it is found that 78% of consumers enjoy ads for product A.

   a) Construct a 90% confidence interval for the proportion of consumers who enjoy ads for product A. *(2 marks)*

   b) Do you think that “more than 75% of all consumers enjoy ads for product A? *(2 marks)*

   c) How many consumers should be interviewed in order to estimate the proportion of consumers who enjoy ads for product A, correct to within 0.02, with probability 0.99? *(2 marks)*

7) A precision instrument is guaranteed to read accurately to within 2 units. A sample of four instrument readings on the same object yielded the measurements 353, 351, 351, and 355.

   a) Find a 90% confidence interval for the population standard deviation. *(3 marks)*

   b) What assumptions are necessary? *(2 marks)*

   c) Does the guarantee seems reasonable? Explain. *(2 marks)*

Bonus Question

Let $Y_1, \cdots, Y_n$ be a random sample from $f_Y(y|\theta)$ with a continuous CDF $F_Y(y|\theta)$. Show that $-\sum lnF_Y(y_i|\theta)$ is a pivotal quantity.