King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

Math 566 (171): Fractional Differential Equations

Midterm Exam

Instructor: Khaled Furati

Duration: 110 minutes

Student Name: ……………………………………………

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1) Consider the Cauchy problem

$$ {}^{c}D^{\frac{7}{5}}\, y + 2\ \, {}^{c}D^{\frac{3}{5}}\, y = 0. $$

$$ y(0) = c, \qquad y'(0) = d. $$

Derive the VIE satisfied by the solution $y \in C^2[0,b]$ and write the successive approximation formula for the solution.

2) Let f be a sufficiently smooth function such that $f(2) = b$, $f'(2) = c$, and ${}_{2}^{c}D_x^{1.4} f|_{x=3} = 4$, find ${}_{2}^{RL}D_x^{1.4} f|_{x=3}$ and ${}_{2}^{GL}D_x^{1.4} f|_{x=3}$.

3) Evaluate the integral

$$ \int_0^1 x^{-1/4}\ \left[{}_{x}I_1^{1/4} x^{1/4} \right]\ dx. $$

4) Consider the equation

$$ D_2^{9/4} y - 2xy = f(x). $$

Provide the appropriate initial conditions when the fractional derivative is the RL derivative and when it is the Caputo derivative.

5) Use series expansion definitions to find (β, ν, γ) such that for any $\alpha > 0$,

$$ I^{\alpha} e^{at} = t^{\beta} E_{\nu,\gamma}(at), \qquad t > 0. $$

6) Show that if $f \in C^1[0,\infty)$, then

$$ I^{1+\alpha} Df(t) = I^{\alpha} f(t) - \frac{f(0)}{\Gamma(\alpha+1)}\ t^{\alpha}, \qquad \alpha > 0. $$

Formula

$$ I^{\alpha} D^{\alpha} f(t) = f(t) - \sum_{k=1}^{n} \frac{D^{\alpha-k} f(0)}{\Gamma(\alpha-k+1)}\ t^{\alpha-k} $$

$$ D^n I_a^{\alpha} = I_a^{n-\alpha}, n > \alpha. $$

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \qquad \int_a^b \phi(x)\ I_a^{\alpha}\psi(x)\ dx \quad = \quad \int_a^b \psi(x)\ I_b^{\alpha}\phi(x)\ dx. $$

Q	1	2	3	4	5	6	Total
Max	10	8	8	8	8	8	50
Points							