King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 521: General Topology

First Exam, Fall Semester 171 (120 minutes)

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Remark: Show *full* details

Q1. (10 points) Show that a topological space (X, τ) is Hausdorff if and only if $\Delta = \{(x, x) \mid x \in X\} \subset X \times X$ is closed.

Q2. (10 points) Let (X, d) be a metric space. Show that for any $A \subset X$ and $x \in X$: there exists a sequence in A converging to x if and only if $x \in \overline{A}$.

Q3. **(20 points)** Show that:
(a) $(0, 1)$ is connected (*Hint:* $\mathbb{R}$ is connected)
(b) $\mathbb{R}^{\mathbb{R}}$ is not metrizable
(c)$f : \mathbb{R}_l \to \mathbb{R}$ is continuous, where

$$f(x) = \begin{cases} x^2 - 1 & x < 0 \\ 1 & x = 0 \\ x^2 + 1 & x > 0 \end{cases}$$

Q4. (30 points) Consider $\mathbb{R}$ with the standard topology.
(a) Find $\overline{\prod_{n \in \mathbb{Z}^+} (-n, n)}$.
(b) Show the $\mathbb{R}^\omega$ is connected.
(c) Show that the uniform topology on $\mathbb{R}^\omega$ is *strictly* coarser that box topology.

Q5. (30 points) Prove or disprove:

(a) $\mathcal{B} = \{[a, b) \mid a, b \in \mathbb{Q} \text{ and } a < b\}$ is a basis for $\mathbb{R}_l$.
(b) If (X, τ) is a topological space and $A \subseteq X$ is connected, then $\bar{A}$ is connected.
(c) If (X, d) is a metric space, then for any $\epsilon > 0$ and

$$B_d(x, \epsilon) := \{y \in X \mid d(x, y) < \epsilon\},$$

the ball with center x and radius ϵ, we have

$$\overline{B_d(x, \epsilon)} = \{y \in X \mid d(x, y) \leq \epsilon\}.$$

Bonus (5 points): Give a metric space (X, d) and a ball of radius r which *strictly contains* some ball of a *strictly bigger* radius $R > r$.

GOOD LUCK