King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 513
Exam I– 2017–2018 (171)
Sunday, October 22, 2017
Allowed Time: 90 minutes

Instructor: **Dr. Boubaker Smii**

Name: ______________________________

ID #: ____________________

Section #: ___________ Serial Number: ___________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification !

Question #	**Grade**	**Maximum Points**
1		**26**
2		**10**
3		**23**
4		**13**
5		**08**
Total:		**80**

Exercise 1:

Let f be the function defined by:

$$f(x) = \begin{cases} 0, & -\pi < x \leq -\frac{\pi}{2} \\ 1, & -\frac{\pi}{2} < x \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x \leq \pi \end{cases} \tag{a}$$

a- Write down the form $S(x)$ of the Fourier series for f.

b- State the precise numerical value of $S(x)$ for each x in the interval $-\pi \leq x \leq \pi$.

c-Compute the Fourier coefficients a_n and b_n, $\forall n \geq 0$ for $f(x)$ and write its Fourier series $S(x)$.

d- Using the fact that $\displaystyle\int_0^x f(t)dt = x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ integrate the Fourier series for $f(x)$ term by term to obtain the series expansion for $\frac{\pi x}{4}$. **(Justify clearly your answer) !**

Exercise 2:

I- The Fourier cosine series for the function $f(x) = 1 - x^2$ on the domain $0 \leq x \leq 1$ is

$$S(x) = \frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos(n\pi x).$$

a- Justify clearly why we can obtain a Fourier series sine ? and write down its expansion.

b- To what values does the Fourier sine - obtained in a- converges ?

Exercise 3:

The Fourier series for the triangular wave $f(x) = \mid x \mid$ on the domain $-\pi \leq x \leq \pi$ is

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \cos((2k-1)x).$$

1- Assume that the Fourier series for $f(x)$ converges uniformly, evaluate the sum

$$S = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

2- Consider the ordinary differential equation $y'' + 25y = f(x)$. (1)

a)- Find the complementary solution to the ODE (1).

b)Find a particular solution to the ODE (1).

c) Write the general solution to the ODE (1).

Exercise 4:

Consider the function $f(x) = \cos x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

1- Find the Fourier Transform $F(w)$ for the function f.

2- Treat the cases $w = 1$ and $w = -1$. (Hint: use the identity: $\cos^2 x = \frac{1+\cos(2x)}{2}$).

Exercise 5:

Given that the Fourier transform of the sign function is given by:

$$F(w) = \begin{cases} \frac{2}{iw}, & w \neq 0 \\ 0, & w = 0 \end{cases} \tag{b}$$

Find the Fourier transform of the (Heaviside) step function $\mathbf{H}(\mathbf{t})$.

(Write your answer in terms of the Dirac delta function δ.)