King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math322 Term(171) Final Exam ***Jan 03, 2018***
Time:180 min.

Name: Sec.:#

Justify all your steps.

Question	**Q1/10**	**Q2/15**	**Q3/15**	**Q4/10**	**Q5/10**	**Q6/15**	**Q7/15**	**Q8/10**
Grades								

Q1. (6+4 points)

a) Use Newton's method to find $\sqrt{2+\sqrt{2}}$ correct to four decimal places.($p_0 = 1.8$)

b) If g has a unique fixed point on the interval [a, b], then bounds for the error involved in using $p_n$to approximate p are given by $|p_n - p| \leq \frac{k^n}{1-k}|p_1 - p_0|$.
Why $0 < k < 1$?

Q2. (5+5+5 points)

a) Approximate $\int_0^2 (sinx)g(x)dx$, where $g(x)$ is given by the table

x	0	0.5	1	1.5	2
g(x)	2	4	3	6	5

Using composite trapezoid rule

b) Suppose that $f(0) = 2, f(2) = B, f(0.5) = 4, f(1) = B, f(1.5) = 3,\ f(3) = 4\ and\ f(\ 2.5) = 3$. Find B if the composite Simpson's rule gives the value **2** for $\int_0^3 f(x)dx$.

c) Analyze the round-off error for the formula

$$f"(x_0) = \frac{f(x_0-h)-2f(x_0)+f(x_0+h)}{h^2} - \frac{h^2}{12}f^{(4)}(c).$$

Then find the optimal *h>0* for the function $f(x) = cosx\ on\ [0.5,\ 1]$ where the data given in 4 decimal places.

Q3. (5+10 points)

a) Show that the initial – value problem has a unique solution.

$$y' = te^{-3t}y, \qquad 0 \leq t \leq 1\,, \qquad y(0) = 0.5$$

b) Use the Runge-Kutta method of order four with N=2 to approximate th IVP in Part a)

Q4. (10 points)

Use Newton backward- difference formula to construct interpolating polynomial of degree three for the following data.

$f(0.1) = -0.6205, f(0.2) = -0.2840, f(0.3) = 0.0066, f(0.4) = -1.0526.$

Approximate $f(0.25)$.

Q5. (10 points)

Consider the function

$$s(x) = \begin{cases} x + 1, & -1 \le x < 1 \\ \frac{1}{6}x^3 + ax^2 + bx + c, & 1 \le x \le 2 \end{cases}$$

Find a choice of the coefficients **a, b, c** such that S(x) as a natural cubic spline function on $[-1,2]$.

Q6. (15 points)

The boundary- value problem

$$y'' = 4(y' - x), \quad 0 \le x \le 1, \quad y(0) = 0, \quad y(1) = 2$$

has the solution $y(x) = \frac{11}{4(1-e^4)}(1 - e^{4x}) - \frac{1}{4}x - \frac{1}{2}x^2$. Use the Finite-Difference method to approximate the solution, and compare the results to the actual solution with $h = 0.25$

Q7. (15 points)

Given the data:

x	1	1.25	1.5	1.75	2
y	5.1	5.79	6.53	7.45	8.46

Construct the least square approximation of the form $y = bx^a$

Q8. (10 points)

Solve by using simplex method

Maximize

$$Z = 3x_1 + 4x_2 + \frac{3}{2}x_3$$

Subject

$$-x_1 - 2x_2 \ge -10$$
$$2x_1 + 2x_2 + x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

The end