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King Fahd University of Petroleum & Minerals (V–1)
Department of Mathematics & Statistics
Math 321 Final Exam
The First Semester of 2017-2018 (171)
Time Allowed: 180 Minutes

Name: ______________________ ID#:______________

Instructor: ______________________ Sec #: __________ Serial #:________

- Mobiles are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		15
2		20
3		20
Total		50

Question #	Answer	Marks
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		70
	Grand Total out of 130	

Q:1 (15 points) Use second order central difference approximations of y' and y'' to write difference equations for the boundary value problem

$y'' + 3y' - 2y = 2x + 3, \quad 0 \leq x \leq 1, \quad y(0) = 2, \quad y(1) = 1$ with $h = 0.2$

Write these difference equations into matrix form.

Q:2 (10+10 points) (a) Let $\{(x_i, y_i), \quad i = 1, 2, 3, \cdots, n\}$ be a given data set of n points. Find the formulas for a_0 and a_1 such that $S_r = \sum_{i=1}^{n} [y_i - a_0 - a_1 x_i]^2$ is minimal.

(b) Use the following data $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0.5 & 2.5 & 3.25 & 3.5 & 5.25 & 5 \end{bmatrix}$ to find a best fit

$y = a_0 + a_1 x$.

Q:3 (8+7 points) (a) Find LU factorization of the matrix A= $\begin{bmatrix} 2 & 2 & 6 & 8 \\ 2 & 3 & 4 & 5 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 1 & 5 \end{bmatrix}$.

(b) Solve the system $LU\bar{x} = b$, where $x = (x_1, x_2, x_3, x_4)$ and $b = (2, 1, 1, 6)$.

Q:1 Using three–point central difference formula, the approximate value of $f''(\frac{\pi}{6})$ for $f(x) = \ln(\sin x)$ with $h = 0.05$ is equal to:

(A) 1.2315

(B) 0.7689

(C) -2.7890

(D) -4.0168

(E) -6.0231

Q:2 The fixed points of $x = 2\sin x$ in the interval $[0, \pi]$ with initial guess $x_o = 1$ and Tolerance $= 10^{-1}$ is approximately equal to:

(A) π

(B) 1.8697

(C) $\pi/4$

(D) 1.0015

(E) 2.3456

Q:3 Using Bisection method, number of iterations necessary to find a root of

$f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$ are:

(A) 5

(B) 6

(C) 7

(D) 8

(E) 10

Q:4 Using the data points, $(6, 3)$, $(7, 5)$, $(8, 1)$ second order interpolating polynomial using Lagrange interpolation or Newton's divided difference interpolation, is:

(A) $-3x^2 + 41x - 135$

(B) $3x^2 + 45x - 135$

(C) $2x^2 + 41x + 145$

(D) $-5x^2 + 41x + 125$

(E) $-2x^2 - 50x - 135$

Q:5 Use 5–points central difference formula to compute $f'(2)$ with $h = 0.1$, for $f(x) = xe^{-x}$.

(A) 0.1234

(B) -0.1525

(C) -0.1353

(D) -0.9765

(E) 0.4563

Q:6 The approximate value of $\int_0^2 e^{x^2}\, dx$ using Simpson's rule is:

(A) 15.4326

(B) 30.5432

(C) 5.5974

(D) 10.3452

(E) 22.1571

Q:7 Using RK–2 Heun method to solve the IVP $y' = xe^{3x} - 2y, \quad 0 \leq x \leq 1, \quad y(0) = 0$ with $h = 0.2$, the value of y_2 is equal to:

(A) 1.2351

(B) 0.1795

(C) 0.2879

(D) 0.4981

(E) 0.3945

Q:8 The linear system
$$\begin{aligned} 4x_1 + x_2 + x_3 &= 4 \\ x_1 + 3x_2 + x_3 &= 6 \\ 2x_1 + 2x_2 + 5x_3 &= 2 \end{aligned}$$

(A) Has infinitely many solutions

(B) Has no solution

(C) Does not converge to a unique solution for some initial guess

(D) Converges to a unique solution for any initial guess

(E) Cannot be solved

Q:9 Using Gauss–Seidel iterative method to solve the linear system

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1 \\ 3x_1 + 6x_2 + 2x_3 &= 0 \\ 3x_1 + 3x_2 + 7x_3 &= 4 \end{aligned}$$

with initial guess $\bar{x} = (0, 0, 0)$, the solution after ONE iteration is:

(A) $(1, 0, 4)$

(B) $(\frac{1}{3}, 0, \frac{2}{7})$

(C) $(\frac{1}{3}, -\frac{1}{6}, \frac{1}{2})$

(D) $(\frac{2}{3}, \frac{1}{6}, \frac{3}{2})$

(E) $(3, 6, 7)$

Q:10 The secant formula to find roots of $x^2 - R = 0$ can be written as:

(A) $x_{i+1} = \dfrac{2x_i^2 + x_i x_{i-1} - R}{x_i + x_{i-1}}$

(B) $x_{i+1} = \dfrac{1}{2}\{x_i + \dfrac{R}{x_i}\}$

(C) $x_{i+1} = \dfrac{x_i x_{i-1}}{x_i + x_{i-1}}$

(D) $x_{i+1} = \dfrac{x_i x_{i-1} - R}{x_i + x_{i-1}}$

(E) $x_{i+1} = \dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$