King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 301 Major Exam 1 (171)

Name:...ID:...............................Sec:.....

Exercise 1: (10 pts) Find the directional derivative of $\varphi(x, y) = x \sin y$ at the point $P_0(1, \frac{\pi}{4})$ in the direction of $P_1(2, \frac{\pi}{3})$.

Exercise 2: (10 pts) Is the vector field $F =< 2xy^3z, 3x^2y^2z, x^2y^3 >$ conservative? If yes, find the potential φ satisfying $\varphi(1, 1, 1) = 2$.

Exercise 3: (14 pts) Evaluate the line integral

$$K = \oint_C (x^2 + y^2)dx + (y - x)dy$$

using Green's theorem where C consists of the boundary of the region in the first quadrant that is bounded by the graphs of $y = x^2$ and $x = y^2$ oriented counterclockwise.

Exercise 4: (16 pts) Find the surface area of that part of the paraboloid $z = x^2 + y^2$ lying in the first quadrant between the planes $z = 0$ and $z = 1$.

Exercise 5: (18 pts) Verify Green's theorem for the vector field $F =< x - y, xy >$ and the curve C is the triangle with vertices $A(2, 1)$, $B(4, 1)$ and $C(2, 5)$,oriented counterclockwise.

Exercise 6: (20 pts) Evaluate $J = \oint_C F \cdot dR$ using Stokes' theorem when the vector field F is $F =< xz, yz, xy >$ and S is the part of the surface $z = 2 - y^2, y \geq 0$, $z \geq 0$, $0 \leq x \leq 1$, oriented upward.

Exercise 7: (12 pts) Evaluate $I = \int\int_S (F \cdot n)ds$ using the divergence theorem, when S is the surface $x^2 + y^2 + z^2 = 1$ oriented outward and F is the vector field
$F =< x^2, -2xy, 3z >$.