King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 280, Final Exam, Term 171

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1. **[4 points]** Find the inverse of the following matrix, if it exists:

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

2. **[4 points]** Let A be an $n \times n$ matrix whose entries consist of $1's\ and\ 0's\ only$. Find $\det(A)$ if each row and each column of A contains one 1 only.
3. **[8 points]** Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2\times2}$.
   a) Show that $W = \{B \in \mathbb{R}^{2\times2}: AB = BA\}$ is a subspace of $\mathbb{R}^{2\times2}$.
   b) Find a basis for W.
4. **[4 points]** Find all real values of a for which $rank(A) = 2$, where

$$A = \begin{bmatrix} 1 & -1 & a/4 \\ 2 & a & -1 \\ 0 & 3 & 8 \end{bmatrix}.$$

5. **[8 points]** Consider the linear transformation $L: \mathbb{P}_3 \to \mathbb{P}_3$ given by

$$L(p(x)) = xp'(x).$$

   a) Find the kernel of L.
   b) Find the range of L.
   c) Find a matrix representing L with respect to the basis $E = \{1, x, x^2\}$ for $\mathbb{P}_3$.
6. **[7 points]** Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \ v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \ v_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}.$$

be vectors in the inner product space $\mathbb{R}^4$ equipped with the standard inner product. Let $S = span\{v_2, v_3\}$.

   a) Find the vector projection of v_1 on S.
   b) Find a basis for $S^{\perp}$, the orthogonal complement of S.

7. **[8 points]** Prove the following statements:
   a) If Q is an $n \times n$ orthogonal matrix, then Q^2 is an orthogonal matrix.
   b) Let A be an $n \times n$ matrix such that $A^2 = 2A$. If λ is an eigenvalue of A, then $\lambda = 0 \ or \ \lambda = 2$.
8. **[8 points]** Use Gram-Schmidt orthogonalization process to generate an orthonormal basis for $\mathbb{R}^3$ starting with the basis

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \right\}.$$

9. **[10 points]** Let

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

   a) Find the eigenvalues of A.
   b) Find the eigenspace corresponding to each eigenvalue of A.
   c) Find a matrix P and a diagonal matrix D such that $P^{-1}AP = D$.
10. **[9 points]** Identify the graph of the quadratic equation

$$9x^2 + 8xy + 3y^2 = 20$$

by first making a suitable substitution that removes the $xy - term$ from the quadratic equation.

All the best,

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