Math201.10, Quiz #2, Term 171

Name: Solutions

ID #:

Serial #:

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1. **[2 points]** Identify (name, vertex(ces), axis) and sketch the graph of $x = 2y^2 - 4y + z^2 + 2$.
2. **[2 points]** Find an equation for the plane through $(1, -2, 4)$ and perpendicular to **both** planes $x + 2y + 3z = 4$ and $4x - 5y - z = 6$.
3. **[4 points]** Let $f(x, y) = \frac{xy}{\sqrt{x^2 - y^2}}$.
   a. Find and sketch the domain of f.
   b. Find an equation for the level curve of f passing through (2,1).
4. **[2 points]** Find the limit: $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$.

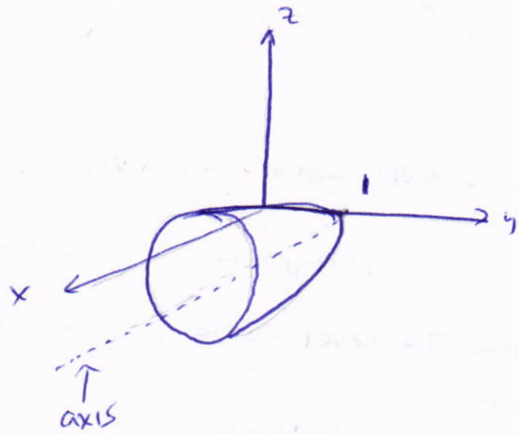
Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ $x = 2(y^2 - 2y + 1) - 2 + z^2 + 2$

$x = 2(y-1)^2 + z^2$

- name: an elliptic paraboloid
- vertex: $(0, 1, 0)$
- axis: the line through $(0,1,0)$ parallel to the x-axis.



$\boxed{2}$ • $\vec{n}_1 = \langle 1, 2, 3 \rangle$, $\vec{n}_2 = \langle 4, -5, -1 \rangle$

• Since the required plane is perpendicular to the given planes, then the normal vector $\vec{n}$ of the required plane is perpendicular to the normal vectors of $\vec{n}_1$ and $\vec{n}_2$ of the given planes. Thus

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & -5 & -1 \end{vmatrix} = \langle 13, 13, -13 \rangle$$

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$\therefore$ a point on the required plane is $(1,-2,4)$.

The equation of the required plane is

$$13(x-1)+13(y+2)-13(z-4)=0$$

$$\Rightarrow \quad (x-1)+(y+2)-(z-4)=0 \qquad 0.5$$

$$\Rightarrow \quad x+y-z=-5$$

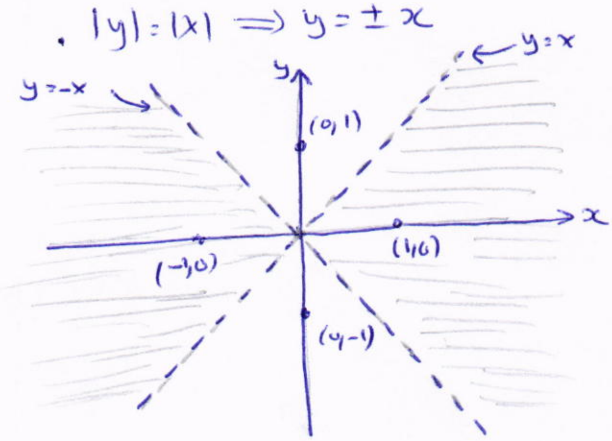
$\boxed{3}$ $f(x,y)=\dfrac{xy}{\sqrt{x^2-y^2}}$

a) Domain $=\{(x,y): x^2-y^2>0\}$ 1

$=\{(x,y): x^2>y^2\}$

$=\{(x,y): |x|>|y|\}$ 0.5

. $|y|=|x| \Rightarrow y=\pm x$



1

b) The level curves are given by

$f(x,y)=c$, c is constant

Since the level curve passes through $(2,1)$, then

$f(2,1)=c$

$$\Rightarrow \quad c=f(2,1)=\frac{2(1)}{\sqrt{4-1}}=\frac{2}{\sqrt3} \qquad 1$$

The equation of the level curve is

$$f(x,y)=\frac{2}{\sqrt3}$$

$$\frac{xy}{\sqrt{x^2-y^2}}=\frac{2}{\sqrt3} \qquad 0.5$$

or $\sqrt3\, xy=2\sqrt{x^2-y^2}$ $\quad (|x|>|y|)$

$\boxed{4}$ $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^3y}{x^6+y^2}$

. along the x-axis : $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}}\frac{x^3y}{x^6+y^2}=\lim_{(x,y)\to(0,0)}\left.\frac{x^3y}{x^6+y^2}\right|_{y=0}$$

$$=\lim_{(x,y)\to(0,0)}\frac{x^3\cdot 0}{x^6+0}=\lim_{(x,y)\to(0,0)}0=0 \qquad 0.5$$

. along $y=x^3$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x^3}}\frac{x^3y}{x^6+y^2}=\lim_{(x,y)\to(0,0)}\left.\frac{x^3y}{x^6+y^2}\right|_{y=x^3}$$

$$=\lim_{(x,y)\to(0,0)}\frac{x^3\cdot x^3}{x^6+x^6}=\lim_{(x,y)\to(0,0)}\frac12=\frac12 \qquad 1$$

Since the limits along the two paths are <u>not equal</u>, then 0.5

$\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^3y}{x^6+y^2}$ <u>does not exist</u>

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Math201.20, Quiz #2, Term 171

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1. [1 points] Identify (name, vertex(ces), axis) and sketch the graph of $z^2 - 2 = x^2 + 2y^2 - 4y - 2x$.
2. [3 points] Find parametric equations for the line of intersection of the two planes $x + 2y + 3z = 4$ and $4x - 5y - z = 6$.
3. [4 points] Let $f(x,y) = \frac{1}{1-\ln(y-x^2)}$.
   a. Find and sketch the domain of f.
   b. Find an equation for and sketch the level curve of f passing through (1,2).
4. [2 points] Find the limit: $\lim_{(x,y)\to(1,1)} \frac{x^2y+xy^2-x-y}{1-\sqrt{xy}}$.

Good luck,

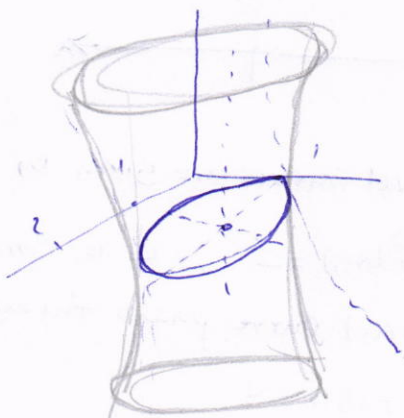
Ibrahim Al-Rasasi

$\boxed{1}$ $z^2 - 2 = x^2 - 2x + 1 + 2(y^2 - 2y + 1) - 1 - 2$

$z^2 - 2 = (x-1)^2 + 2(y-1)^2 - 3$

$(x-1)^2 + 2(y-1)^2 - z^2 = 1$ 0.5

- name: a hyperboloid of one sheet 0.5
- axis: a line parallel to the z-axis and through (1,1).



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2 $\vec{n}_1 = \langle 1,2,3\rangle$, $\vec{n}_2 = \langle 4,-5,-1\rangle$

. a parallel vector to the line $= \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & -5 & -1 \end{vmatrix} = \langle 13, 13, -13\rangle$

1.5

. a point on the line:

Set $y=0$. Then we get

$$\begin{cases} x+3z=4 \\ 4x-z=6 \end{cases} \Longrightarrow x = \frac{22}{13}, \; z = \frac{10}{13}$$

a point is $\left(\frac{22}{13}, 0, \frac{10}{13}\right)$ 1

Other points
$\left(0, \frac{-22}{13}, \frac{32}{13}\right)$,
$\left(\frac{32}{13}, \frac{10}{13}, 0\right)$

. Parametric equations of the line:

$$x = \frac{22}{13} + 13t$$
$$y = 13t \qquad , t \in \mathbb{R}$$
$$z = \frac{10}{13} + 13t$$

0.5

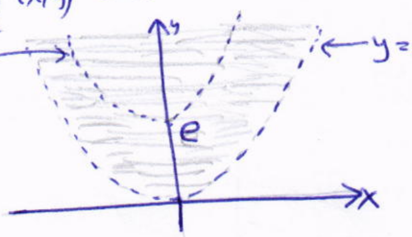
3 $f(x,y) = \dfrac{1}{1-\ln(y-x^2)}$

a) Domain $= \{(x,y) : y - x^2 > 0 \text{ and } 1 - \ln(y-x^2) \neq 0\}$

$= \{(x,y) : y > x^2 \text{ and } \ln(y-x^2) \neq 1\}$

$= \{(x,y) : y > x^2 \text{ and } y - x^2 \neq e\}$

$= \{(x,y) : y > x^2 \text{ and } y \neq x^2 + e\}$ 2



1

b) The level curves are given by

$f(x,y) = c$, c is constant

Since the level curve passes through $(1,2)$, then $f(1,2) = c$

$\Longrightarrow c = f(1,2) = \dfrac{1}{1-\ln 1} = \dfrac{1}{1-0} = 1$

The equation of the level curve is

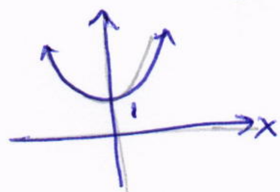
$f(x,y) = 1$

$\Longrightarrow \dfrac{1}{1-\ln(y-x^2)} = 1$

$\Longrightarrow 1 - \ln(y-x^2) = 1$

$\Longrightarrow \ln(y-x^2) = 0$

$\Longrightarrow y - x^2 = 1 \Longrightarrow \boxed{y = x^2 + 1}$ 1



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$$\lim_{(x,y)\to(1,1)} \frac{x^2y + yx^2 - x - y}{1-\sqrt{xy}} \cdot \frac{1+\sqrt{xy}}{1+\sqrt{xy}} =$$

$$\lim_{(x,y)\to(1,1)} \frac{(x^2y + yx^2 - x - y)}{1 - xy} \cdot (1+\sqrt{xy})$$ 1

. $x^2y + yx^2 - x - y$
$= xy(x+y) - (x+y)$
$= (x+y)(xy-1)$ 0.5

$$= \lim_{(x,y)\to(1,1)} \frac{(x+y)(xy-1)}{1-xy} \cdot (1+\sqrt{xy})$$

$$= \lim_{(x,y)\to(1,1)} -(x+y)(1+\sqrt{xy})$$

$$= -(1+1)(1+\sqrt{1\cdot 1})$$

$$= -4$$

0.5