King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 101(45) Class Test I Fall 2017(171)

ID#:________________ NAME:________________________________

(1) Evaluate the limit, if it exists:

(a) $\lim_{x\to 0^-} (x-7)\frac{2x}{|3x|}$

(b) $\lim_{x\to -\infty} (-2x-1)^3(x-1)^2(-x+2)$

(c) $\lim_{x\to m} \frac{1}{x-m}$, where m is a positive integer.

(d) $\lim_{x\to 1^+} \frac{|x^2-3x+2|}{x^2-1}$.

(e) $\lim_{x\to 0} \frac{x^3-x^2}{\sqrt{1+2x^2}-1}$.

(f) $\lim_{x\to\frac{1}{4}}(\frac{4}{4x-1} - \frac{5}{4x^2+3x-1})$.

(g) $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$.

(h) $\lim_{x\to e}[(x-e)\cos(\frac{\sqrt{x+2}}{x-e})]$.

(i) $\lim_{x\to +\infty}(\sqrt{x^2+3}-x)$.

(j) $\lim_{x\to -1}[[\frac{1}{2}x-1]]$, where [[.]] denotes the greatest integer function.

(2) Use the Intermediate Value Theorem to show that the equation $x^4 + x^2 = 1$ has a solution.

(3) Use the graph of $f(x) = \frac{1}{x}$ to find a number δ such that $|\frac{1}{x} - \frac{1}{3}| < \frac{1}{5}$ whenever $|x - 3| < \delta$.

(4) Let $f(x) = 3x - 5$. Find the largest value of δ such that $|f(x) - 4| < 0.01$ whenever $|x + 3| < \delta$.

(5) Sketch a graph of a function $f(x)$ that satisfies the following conditions:

1. $\lim_{x \to 1^+} f(x) = 0$;
2. $\lim_{x \to 1^-} f(x) = 2$;
3. $\lim_{x \to +\infty} f(x) = 2$;
4. $\lim_{x \to -\infty} f(x) = -1$;
5. $\lim_{x \to -2} f(x) = +\infty$;
6. $f(0) = 0$ and $f(1) = 1$.

(6) Consider the function $f(x) = \frac{2}{\sqrt{4-x}}$ and the point $P(0, f(0))$.

(i) Find the instantaneous rate of change of $f(x)$ with respect to x.

(ii) Find the slope of the graph of $y = f(x)$ at the point P.

(7) Given that $f(x) = \begin{cases} x^2 - 1 & -1 \leq x \leq 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x \leq 2 \\ 0 & 2 < x \leq 3 \end{cases}$

Find all points in **[0,3]** where f is discontinuous. Determine if the discontinuity is removable.

(8) Let $f(x) = \frac{2-x}{\sqrt{x^2-4}}$. Using the concept of limit, find

(a) all horizontal asymptotes (if any)

(b) all vertical asymptotes (if any)

(9) Answer TRUE($\surd$) or FALSE ($\times$)

(a) The function $f(x) = \ln x$ is differentiable everwhere.

(b) $\lim\limits_{x\to 4}(\frac{2x}{x-4} - \frac{8}{x-4}) = \lim\limits_{x\to 4}\frac{2x}{x-4} - \lim\limits_{x\to 4}\frac{8}{x-4}$.

(c) If $\lim\limits_{x\to 6} f(x)g(x)$ exists, then the limit must be $f(6)g(6)$.

(d) If the line $x = 1$ is a vertical asymptote, then f is not defined at 1.

(e) If f is continuous on $[-1, 1]$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number m such that $|m| < 1$ and $f(m) = \pi$.

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