King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 33 Quiz IV (B) (Term 171)

Name : ....KEX.... ID #.................................... Serial #: ....

1. The sum of all critical numbers of the function

$D_f: \mathbb{R} - \{-1\}$

$$f(x) = \frac{(x-4)^2}{\sqrt[3]{x+1}}$$

is

$$f'(x) = \frac{2(x-4)\sqrt[3]{x+1} - \frac{1}{3}(x+1)^{-2/3}(x-4)^2}{(x+1)^{2/3}}$$

$$= \frac{(x-4)(x+1)^{-2/3}\left[2(x+1) - \frac{1}{3}(x-4)\right]}{(x+1)^{2/3}} = 0$$

a) -2
b) -1
c) 1
d) 2
e) 4

$\Rightarrow x = 4$ or $\frac{5}{3}x + \frac{10}{3} = 0 \Rightarrow x = \frac{-10}{3} \cdot \frac{3}{5} = -2$

$\Rightarrow$ Sum $= 4 - 2 = 2$

2. The absolute maximum of $f(x) = \dfrac{\ln x}{x^2}$ on the interval $[1, e]$ is

a) $\dfrac{1}{2}$
b) $\dfrac{1}{e^2}$
c) 1
d) $\dfrac{1}{2e}$
e) e

$$f'(x) = \frac{x - 2x\ln x}{x^4} = 0$$

$\Rightarrow x(1 - 2\ln x) = 0$

$\Rightarrow 1 - 2\ln x = 0$ or $x = 0$ ✗

$\Rightarrow -2\ln x = -1$

$\Rightarrow \ln x = \frac{1}{2} \Rightarrow x = e^{1/2}$

$f(1) = 0$

$f(e^{1/2}) = \frac{1}{2} \cdot \frac{1}{e} = \frac{1}{2e}$

$f(e) = \frac{1}{e^2}$

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3. $\sin^{-1} x + \cos^{-1} x =$

a) $\frac{\pi}{2}$

b) 1

c) $\frac{\pi}{3}$

d) 0

e) None of these

$f(x) = \sin^{-1} x + \cos^{-1} x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$

$\Rightarrow f(x) = C$

Let $x=0 \Rightarrow \sin^{-1}(0) + \cos^{-1}(0) = \frac{\pi}{2}$

4. $\lim_{x\to\infty} \sqrt[x]{x} =$

a) $-\infty$

b) 0

c) 1

d) e

e) ∞

$f(x) = x^{\frac{1}{x}}$ Take ln

$\Rightarrow \ln f(x) = \frac{1}{x} \ln x$

Take $\lim_{x\to\infty}$ to get

$\lim_{x\to\infty} \frac{\ln x}{x} \overset{LHS}{=} \lim_{x\to\infty} \frac{1}{x} = 0$

$\Rightarrow \lim_{x\to\infty} \sqrt[x]{x} = e^0 = 1$

5. The slant asymptote of $f(x) = e^x + x + 1$ is

Since $\lim_{x \to -\infty} \left(e^x + x + 1 - (x+1)\right) = 0$

$\Rightarrow y = x + 1$ is a slant Asymptote

a) $y = x$
b) $y = -2x + 1$
c) $y = 2x + 1$
d) $y = x + 1$
e) None of these

6. The function $f(x) = \ln(x^2 - 3x + 2)$ has

D_f: $x^2 - 3x + 2 > 0$

$\Rightarrow (x-2)(x-1) > 0$

$+ \quad 1 \quad - \quad 2 \quad +$

$(-\infty, 1) \cup (2, \infty)$

a) neither local minimum nor local maximum
b) no local minimum and one local maximum
c) two local minima and one local maximum
d) one local minimum and two local maxima
e) one local minimum and one local maximum

$f'(x) = \dfrac{2x-3}{x^2-3x+2} = 0 \Rightarrow x = 3/2 \notin D_f$

Also, $x^2 - 3x + 2 \neq 0$

7. The graph of the function

$$f(x) = -2x - 4\sin x, \; 0 \le x \le 2\pi$$

is decreasing on the interval(s)

$f'(x) = -2 - 4\cos x = 0$

$\Rightarrow \cos x = -\frac{1}{2}$

$\Rightarrow x = \frac{2\pi}{3}$ & $x = \frac{4\pi}{3}$

$-$ $\quad$ $+$ $\quad$ $-$

$0 \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad 2\pi$

a) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

b) $\left(0, \frac{4\pi}{3}\right)$

c) $\left(\frac{\pi}{3}, \pi\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$

d) $\left(\frac{2\pi}{3}, \pi\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

e) $\left(0, \frac{2\pi}{3}\right)$ and $\left(\frac{4\pi}{3}, 2\pi\right)$

8. The sum of all numbers c that satisfy the conclusion of Rolle's Theorem for the function $f(x) = \dfrac{1 - \sin x}{1 + \sin x}$ on the interval $[0, \pi]$ is

$\sin x \ne -1$ check

$f(0) = 1 = f(\pi)$

$\Rightarrow f'(c) = \dfrac{-\cos c\,(1+\sin c) - \cos c\,(1-\sin c)}{1+\sin c} = 0$

$\Rightarrow \cos c = 0 \Rightarrow c = \frac{\pi}{2}$

a) Rolle's Theorem is not applicable

b) $\frac{\pi}{2}$

c) π

d) $\frac{3\pi}{2}$

e) $\frac{3\pi}{4}$

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9. $\lim_{x\to 1}\left(\frac{x}{\ln x} - \frac{1}{x\ln x}\right) =$

$\lim_{x\to 1}\left(\frac{x^2-1}{x\ln x}\right) \overset{LHS}{=} \lim_{x\to 1}\frac{2x}{1+\ln x}$

$= 2$

a) 0
b) 2
c) 1
d) ∞
e) 3

10. If the function $f(x) = x^3 + 2ax^2 - 3bx + 1$ has an inflection point at $(1,2)$, then $2a + b^3$ equals

$f'(x) = 3x^2 + 4ax - 3b$

$f''(x) = 6x + 4a$

$f''(1) = 0 \Rightarrow 6 + 4a = 0 \Rightarrow a = -3/2$

Also, $f(1) = 2 \Rightarrow 1 + 2a - 3b + 1 = 2$

$\Rightarrow -3 - 3b = 0 \Rightarrow b = -1$

$\Rightarrow 2a + b^3 = -3 - 1 = -4$

a) -1
b) 3
c) 2
d) -2
e) -4