

King Fahd University of Petroleum and Minerals  
 Department of Mathematics and Statistics  
 Math 101 Section 33 Quiz IV (B) (Term 171)

Name: KEY ID #: ..... Serial #: .....

1. The sum of all critical numbers of the function

$$f(x) = \frac{(x-4)^2}{\sqrt[3]{x+1}}$$

$$D_f: \mathbb{R} - \{-1\}$$

is

$$f'(x) = \frac{2(x-4)\sqrt[3]{x+1} - \frac{1}{3}(x+1)^{-2/3}(x-4)^2}{(x+1)^{4/3}}$$

a) -2

b) -1

c) 1

d) 2

e) 4

$$= \frac{(x-4)(x+1) \left[ 2(x+1) - \frac{1}{3}(x-4) \right]}{(x+1)^{4/3}} = 0$$

$$\Rightarrow x=4 \text{ or } \frac{5}{3}x + \frac{10}{3} = 0 \Rightarrow x = -\frac{10}{3} + \frac{3}{5} = -2$$

$$\Rightarrow \text{Sum} = 4 - 2 = 2$$

2. The absolute maximum of  $f(x) = \frac{\ln x}{x^2}$  on the interval  $[1, e]$  is

a)  $\frac{1}{2}$

b)  $\frac{1}{e^2}$

c) 1

d)  $\frac{1}{2e}$

e) e

$$f'(x) = \frac{x - 2x \ln x}{x^4} = 0$$

$$\Rightarrow x(1 - 2 \ln x) = 0$$

$$\Rightarrow 1 - 2 \ln x = 0 \text{ or } x = 0 \text{ X}$$

$$\Rightarrow -2 \ln x = -1$$

$$\Rightarrow \ln x = \frac{1}{2} \Rightarrow x = e^{1/2}$$

$$f(1) = 0$$

$$f(e^{1/2}) = \frac{1}{2} \cdot \frac{1}{e} = \frac{1}{2e}$$

$$f(e) = \frac{1}{e^2}$$

3.  $\sin^{-1} x + \cos^{-1} x =$

a)  $\frac{\pi}{2}$

b) 1

c)  $\frac{\pi}{3}$

d) 0

e) None of these

$$f(x) = \sin^{-1} x + \cos^{-1} x$$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow f(x) = C$$

$$\text{let } x=0 \Rightarrow \sin^{-1}(0) + \cos^{-1}(0) = \frac{\pi}{2}$$

4.  $\lim_{x \rightarrow \infty} \sqrt[x]{x} =$

a)  $-\infty$

b) 0

c) 1

d) e

e)  $\infty$

$$f(x) = x^{\frac{1}{x}} \quad \text{Take } \ln$$

$$\Rightarrow \ln f(x) = \frac{1}{x} \ln x$$

Take  $\lim_{x \rightarrow \infty}$  to get

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\Rightarrow \lim_{x \rightarrow \infty} \sqrt[x]{x} = e^0 = 1$$

5. The slant asymptote of  $f(x) = e^x + x + 1$  is

- a)  $y = x$
- b)  $y = -2x + 1$
- c)  $y = 2x + 1$
- d)  $y = x + 1$
- e) None of these

Since  $\lim_{x \rightarrow -\infty} (e^x + x + 1 - (x + 1)) = 0$

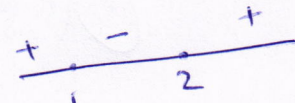
$\Rightarrow y = x + 1$  is a slant Asymptote

6. The function  $f(x) = \ln(x^2 - 3x + 2)$  has

- a) neither local minimum nor local maximum
- b) no local minimum and one local maximum
- c) two local minima and one local maximum
- d) one local minimum and two local maxima
- e) one local minimum and one local maximum

$D_f: x^2 - 3x + 2 > 0$

$\Rightarrow (x-2)(x-1) > 0$



$(-\infty, 1) \cup (2, \infty)$

$f'(x) = \frac{2x-3}{x^2-3x+2} = 0 \Rightarrow x = 3/2 \notin D_f$

Also,  $x^2 - 3x + 2 \neq 0$

7. The graph of the function

$$f(x) = -2x - 4 \sin x, \quad 0 \leq x \leq 2\pi$$

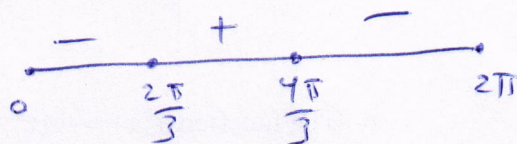
is decreasing on the interval(s)

- a)  $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$
- b)  $\left(0, \frac{4\pi}{3}\right)$
- c)  $\left(\frac{\pi}{3}, \pi\right)$  and  $\left(\frac{5\pi}{3}, 2\pi\right)$
- d)  $\left(\frac{2\pi}{3}, \pi\right)$  and  $\left(\frac{3\pi}{2}, 2\pi\right)$
- e)  $\left(0, \frac{2\pi}{3}\right)$  and  $\left(\frac{4\pi}{3}, 2\pi\right)$

$$f'(x) = -2 - 4 \cos x = 0$$

$$\Rightarrow \cos x = -\frac{1}{2}$$

$$\Rightarrow x = \frac{2\pi}{3} \text{ and } x = \frac{4\pi}{3}$$



8. The sum of all numbers  $c$  that satisfy the conclusion of Rolle's Theorem for the function  $f(x) = \frac{1 - \sin x}{1 + \sin x}$  on the interval  $[0, \pi]$  is

$\sin x \neq -1$  check

a) Rolle's Theorem is not applicable

b)  $\frac{\pi}{2}$

c)  $\pi$

d)  $\frac{3\pi}{2}$

e)  $\frac{3\pi}{4}$

$$f(c) = 1 = f(\pi)$$

$$\Rightarrow f'(c) = \frac{-\cos c (1 + \sin c) - \cos c (1 - \sin c)}{1 + \sin c} = 0$$

$$\Rightarrow \cos c = 0 \Rightarrow c = \frac{\pi}{2}$$

$$9. \lim_{x \rightarrow 1} \left( \frac{x}{\ln x} - \frac{1}{x \ln x} \right) =$$

$$\lim_{x \rightarrow 1} \left( \frac{x^2 - 1}{x \ln x} \right)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{2x}{1 + \ln x}$$

$$= 2$$

- a) 0
- b) 2
- c) 1
- d)  $\infty$
- e) 3

10. If the function  $f(x) = x^3 + 2ax^2 - 3bx + 1$  has an inflection point at  $(1, 2)$ , then  $2a + b^3$  equals

$$f'(x) = 3x^2 + 4ax - 3b$$

$$f''(x) = 6x + 4a$$

$$f''(1) = 0 \Rightarrow 6 + 4a = 0 \Rightarrow a = -3/2$$

$$\text{Also, } f(1) = 2 \Rightarrow 1 + 2a - 3b + 1 = 2$$

$$\Rightarrow -3 - 3b = 0 \Rightarrow b = -1$$

$$\Rightarrow 2a + b^3 = -3 - 1 = -4$$

- a) -1
- b) 3
- c) 2
- d) -2
- e) -4