King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 33 Quiz III (Term 171)

Name : KEY ID #.................................... Serial #: ....

6pts 1. If $y = u^2 - 1$ and $u = e^{2x} + \ln x$, then find $\frac{dy}{dx}$ when $x = 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{2pts}$$

$$= (2u)\left(2e^{2x} + \frac{1}{x}\right) \quad \text{1pt}$$

$$= 2e^2(2e^2+1) = 4e^4 + 2e^2 \quad \text{1pt}$$

If $x = 1$, then $u = e^2$ 1pt

Another Solution $y = (e^{2x} + \ln x)^2 - 1 \Rightarrow \frac{dy}{dx} = 2(e^{2x} + \ln x)(2e^{2x} + \frac{1}{x})$ 1pt 3pts

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 2e^2(2e^2+1) = 4e^4 + 2e^2 \quad \text{2pts}$$

6pts 2. Let $h(x) = 2g(x) + f(\sqrt{g(x)})$ and $h'(-1) = 7$, $f'(3) = 18$, $g(-1) = 9$, then find $g'(-1)$.

$$h'(x) = 2g'(x) + f'(\sqrt{g(x)}) \cdot \frac{g'(x)}{2\sqrt{g(x)}} \quad \text{3pts}$$

$$\Rightarrow h'(-1) = 2g'(-1) + f'(\sqrt{g(-1)}) \cdot \frac{g'(-1)}{2\sqrt{g(-1)}} \quad \text{1pt}$$

$$\Rightarrow 7 = 2g'(-1) + f'(3) \cdot \frac{g'(-1)}{2(3)}$$

$$\Rightarrow 7 = 2g'(-1) + 3g'(-1) \Rightarrow g'(-1) = \frac{7}{5} \quad \text{2pts}$$

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5 pts 3. Find the slope of the tangent line to the graph of $x^2y + y^4 = 4 + 2x$ at the point $(-1, 1)$.

Diff. w.r.t. x to get

$2xy + x^2y' + 4y^3y' = 2$ 3 pts

Substitute to get

$-2 + y' + 4y' = 2 \Rightarrow 5y' = 4 \Rightarrow y' = \frac{4}{5}$ 1 pt

1 pt

5 pts 4. For $t > 0$, find $\frac{d}{dt}\left[\sin^{-1}\left(\frac{t-4}{t+4}\right)\right]$ in its simplest form.

$= \frac{1}{\sqrt{1-\left(\frac{t-4}{t+4}\right)^2}} \cdot \frac{1(t+4)-(t-4)}{(t+4)^2}$ 3 pts

$= \frac{t+4}{\sqrt{(t+4)^2-(t-4)^2}} \cdot \frac{8}{(t+4)^2}$

$= \frac{8}{4(t+4)\sqrt{t}} = \frac{2}{(t+4)\sqrt{t}} = \frac{2\sqrt{t}}{t(t+4)}$ 2 pts

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4 pts 5. If $f(x) = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$, then find $f'(2)$

$$f'(x) = \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{x/2}{1+x^2/4}$$ 3 pts

$$\Rightarrow f'(2) = \frac{4}{8} - \frac{\pi}{4} - \frac{1}{2} = -\frac{\pi}{4}$$ 1 pt

4 pts 6. If $y = (1+\sqrt{x})^x$, then find $g'(1)$

$$\Rightarrow \ln y = x\ln(1+\sqrt{x})$$

$$\Rightarrow \frac{y'}{y} = \ln(1+\sqrt{x}) + \frac{x \cdot \frac{1}{2\sqrt{x}}}{1+\sqrt{x}}$$ 2 pts

If $x=1$, then $y=2$

$$\Rightarrow y' = 2\left[\ln 2 + \frac{1}{4}\right] = 2\ln 2 + \frac{1}{2}$$ 2 pts

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5pts 7. Let $f(x) = 1 + 2x - x^2$, $x \le 1$. Then find $\frac{df^{-1}}{dx}|_{x=-2}$ $= \dfrac{1}{f'(f^{-1}(-2))}$ 1pt

$f'(x) = 2 - 2x$ 1pt.

To find $f^{-1}(-2)$, we need to solve

$-2 = 1 + 2x - x^2 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1$ ✓ or $x = 3$ X $\Rightarrow f^{-1}(-2) = -1$ 2pts

$\Rightarrow \frac{df^{-1}}{dx}\Big|_{x=-2} = \dfrac{1}{f'(1)} = \dfrac{1}{2+2} = \dfrac{1}{4}$ 1pt

5pts 8. Let $f(x) = x^4 e^x$. Then find $f^{(4)}(x)$ at $x = 0$.

$f'(x) = 4x^3 e^x + x^4 e^x$ 1pt

$f''(x) = 12x^2 e^x + 4x^3 e^x + 4x^3 e^x + x^4 e^x = 12x^2 e^x + 8x^3 e^x + x^4 e^x$ 1pt

$f'''(x) = 24x e^x + 12x^2 e^x + 24x^2 e^x + 8x^3 e^x + 4x^3 e^x + x^4 e^x$

$= 24x e^x + 36x^2 e^x + 12x^3 e^x + x^4 e^x$ 1pt

$f^{(4)}(x) = 24e^x + 24x e^x + 72x e^x + 36x^2 e^x + 36x^2 e^x + 12x^3 e^x + 4x^3 e^x + x^4 e^x$ 1pt

$\Rightarrow f^{(4)}(0) = 24$ 1pt