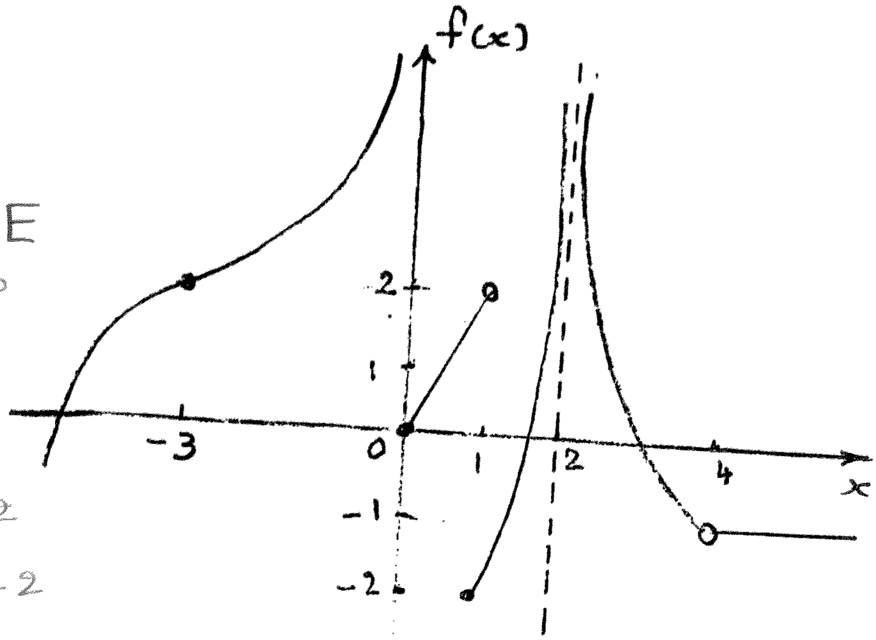
King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 3 Quiz I (Term 171)

Name : .... ID #.................................. Serial #: ....

1. Use the graph of $y = f(x)$ in the figure to evaluate the limit if it exists. If it does not exist, explain why? Use the symbols ∞ or $-\infty$ when needed.

1

(i) $\lim_{x\to-3} f(x) = 2$



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(ii) $\lim_{x\to0} f(x) = \text{DNE}$

$\lim_{x\to0^-} f(x) = \infty$

$\lim_{x\to0^+} f(x) = 0$

3

(iii) $\lim_{x\to1} f(x) = \text{DNE}$

$\lim_{x\to1^-} f(x) = 2$

$\lim_{x\to1^+} f(x) = -2$

3

(iv) $\lim_{x\to2} f(x) = \infty$

$\lim_{x\to2^-} f(x) = \infty = \lim_{x\to2^+} f(x)$

1

(v) $\lim_{x\to4} f(x) = -1$

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(vi) $\lim_{x\to4^-} (2f(x) + 3g(x))$ where $g(x) = [x-1]$ and $[x]$ is the greatest integer $\le x$.

$= 2\lim_{x\to4^-} f(x) + 3\lim_{x\to4^-} [x-1]$

$= 2(-1) + 3(2) = 4$

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2. Evaluate the limit if it exists. If it does not exist, explain why? Use the symbols ∞ or $-\infty$ when needed.

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(i) $\lim_{x\to-6^+} \frac{|x^2+5x-6|}{x+6}$ $= \lim_{x\to-6^+} \frac{|(x+6)(x-1)|}{x+6}$

$= \lim_{x\to-6^+} \frac{|x+6|\,|x-1|}{x+6}$

Note as $x \to -6^+$
$x+6>0$
So $|x+6| = x+6$

$= \lim_{x\to-6^+} \frac{\cancel{(x+6)}\,|x-1|}{\cancel{(x+6)}} = |-6-1| = 7$

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(ii) $\lim_{x\to 0} \left[\frac{1}{1+10\sin x}\right]$ where $[x]$ is the greatest integer less than or equal to x.

$\lim_{x\to 0^+} \left[\frac{1}{1+10\sin x}\right] = 0$ since $0 < \frac{1}{1+10\sin x} < 1$

$\lim_{x\to 0^-} \left[\frac{1}{1+10\sin x}\right] = 1$ since $2 > \frac{1}{1+10\sin x} > 1$

$\therefore \lim_{x\to 0} \left[\frac{1}{1+10\sin x}\right]$ DNE

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(iii) $\lim_{x\to 1} f(x)$ where

$$f(x) = \begin{cases} |x+1| & \text{if } x \le 1 \\ 2x & \text{if } x > 1 \end{cases}$$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} |x+1| = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x) = 2.$

$\therefore \lim_{x\to 1} f(x) = 2$

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5 (iv) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-\sqrt{x}}$ $= \lim_{x\to 1} \frac{\cancel{(\sqrt{x}-1)}}{\sqrt{x}\cancel{(\sqrt{x}-1)}} = 1$

3 (v) $\lim_{u\to -1} \frac{u^3+1}{u^2-1}$ $= \lim_{u\to -1} \frac{\cancel{(u+1)}(u^2-u+1)}{\cancel{(u+1)}(u-1)} = \frac{1+1+1}{-1-1} = -\frac{3}{2}$

4 (vi) $\lim_{x\to 0} \sin^2 x \cos \frac{\pi}{x}$: By the squeeze Theorem:

$-1 \le \cos\frac{\pi}{x} \le 1$

$\therefore \quad -\sin^2 x \le \sin^2 x \cos\frac{\pi}{x} \le \sin^2 x.$

Since $\lim_{x\to 0}(-\sin^2 x) = \lim_{x\to 0} \sin^2 x = 0 \Rightarrow \lim_{x\to 0} \sin^2 x \cos\frac{\pi}{x} = 0$

3 (vii) $\lim_{h\to 0^+} \frac{(2+h)^3-8}{h^2}$

$= \lim_{h\to 0^+} \frac{(\cancel{8} + 12h + 6h^2 + h^3) - \cancel{8}}{h^2}$

$= \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{h^{\cancel{2}}} = \infty$

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3. If $\lim_{x\to 0} \frac{\sqrt{ax+b}-4}{3x} = 2$, then find the values of a and b.

$$\lim_{x\to 0}\left(\sqrt{ax+b}-4\right) = \lim_{x\to 0}\left(\frac{\sqrt{ax+b}-4}{3x}\right)(3x) = 2(0) = 0.$$

$$\therefore \lim_{x\to 0}\sqrt{ax+b} = 4. \quad \text{OR} \quad \sqrt{0+b} = 4 \Rightarrow \boxed{b=16}.$$

<u>Now</u>: $2 = \lim_{x\to 0}\frac{\sqrt{ax+16}-4}{3x}\cdot\frac{\sqrt{ax+16}+4}{\sqrt{ax+16}+4}$

$$= \lim_{x\to 0}\frac{ax+\cancel{16}-\cancel{16}}{3x\left(\sqrt{ax+16}+4\right)}$$

$$= \lim_{x\to 0}\frac{a}{3\left(\sqrt{ax+16}+4\right)} = \frac{a}{3\left(\sqrt{16}+4\right)} = \frac{a}{24}$$

$$\Rightarrow \boxed{a = 48}$$

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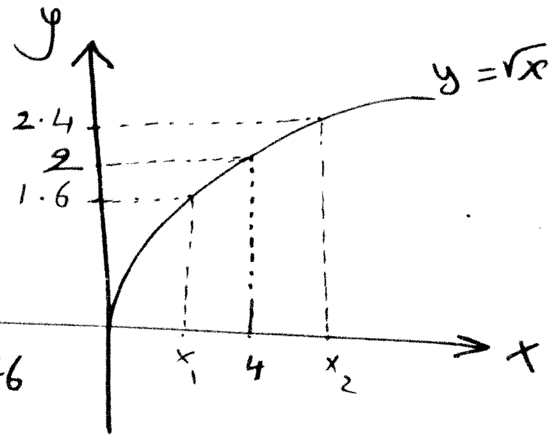
4. Use the graph of $f(x)=\sqrt{x}$ to find the largest δ such that if $|x-4|<\delta$ then $|\sqrt{x}-2|<0.4$

Note that $\epsilon = 0.4$



$$\sqrt{x_1} = 1.6 \Rightarrow x_1 = 1.6^2 = 2.56$$

$$\sqrt{x_2} = 2.4 \Rightarrow x_2 = 2.4^2 = 5.76$$

$\therefore$ The max $\delta = \min\{x_2-4,\ 4-x_1\}$

$$= \min\{5.76-4,\ 4-2.56\}$$

$$= \min\{1.76,\ 1.44\}$$

$$= 1.44$$

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