King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Exam II
Term 171
Tuesday 28/11/2017
Net Time Allowed: 120 minutes

MASTER VERSION

Q	MM	V1	V2	V3	V4
1	a	d	c	c	e
2	a	d	c	a	e
3	a	e	b	d	a
4	a	c	c	a	a
5	a	b	e	c	a
6	a	c	d	e	d
7	a	a	a	c	a
8	a	b	a	d	e
9	a	d	c	c	e
10	a	e	e	e	e
11	a	d	a	b	d
12	a	d	d	c	d
13	a	a	b	b	c
14	a	a	e	e	b
15	a	c	a	c	d
16	a	c	e	c	c
17	a	b	d	b	a
18	a	d	a	a	c
19	a	d	a	d	c
20	a	b	c	a	b

1. If $f(x) = \cot x \cdot \csc x$, then $f'\left(\frac{\pi}{4}\right) =$

(a) $-3\sqrt{2}$

(b) $3\sqrt{2}$

(c) $2\sqrt{2}$

(d) $-2\sqrt{2}$

(e) $\sqrt{2}$

$f'(x) = \cot x(-\csc x \cdot \cot x) + \csc x \cdot (-\csc^2 x)$
$= -\csc x \cdot \cot^2 x - \csc^3 x$

$f'(\pi/4) = -\sqrt{2} \cdot 1 - (\sqrt{2})^3$
$= -\sqrt{2} - 2\sqrt{2} = -3\sqrt{2}$

2. If $f(x) = \ln(3x^2 + x + 1)$, then $f'(0) =$

(a) 1

(b) 0

(c) 2

(d) 3

(e) -1

$f'(x) = \dfrac{6x+1}{3x^2+x+1}$; $f'(0) = 1$

3. Consider the equation $(y^2-1)\,x^2+y^3=\dfrac{1}{x}$. What is the value of y' at $y=-1$?

(a) -1

(b) -2

(c) 0

(d) 1

(e) 2

$(y^2-1)(2x)+x^2(2yy')+3y^2y'=-\dfrac{1}{x^2}$

at $y=-1 \Rightarrow -1=\dfrac{1}{x} \Rightarrow x=-1$

therefore: $(0)(-2)+(1)(-2y')+3y'=-1$

$y'=-1$

4. If $x^2+xy+y^2=1$, then $y''=$

(a) $-6/(x+2y)^3$

(b) $(2x+y)/(x+2y)^2$

(c) $-3x/(x+2y)^2$

(d) $2/(x+2y)^3$

(e) $(x^2+y^2)/(x+2y)^3$

$2x+xy'+y+2yy'=0$

$y'(x+2y)=-(2x+y)$

$y'=-\dfrac{2x+y}{x+2y}$

$y''=-\dfrac{(2+y')(x+2y)-(1+2y')(2x+y)}{(x+2y)^2}$

$y''=-\dfrac{\cancel{2x}+4y+y'x+\cancel{2yy'}-\cancel{2x}-y-4xy'-\cancel{2yy'}}{(x+2y)^2}$

$y''=-\dfrac{3y-3xy'}{(x+2y)^2}$

$y''=-\dfrac{3y+3x\cdot\dfrac{(2x+y)}{x+2y}}{(x+2y)^2}$

$y''=-\dfrac{3y(x+2y)+3x(2x+y)}{(x+2y)^3}=$

$\Rightarrow$

$y''=-\dfrac{6x^2+6y^2+6xy}{(x+2y)^3}=-\dfrac{6}{(x+2y)^3}$

5. Find the value of the constant c such that $y = \frac{3}{2}x + 6$ is tangent to $y = c\sqrt{x}$:

(a) 6

(b) 4

(c) -4

(d) -6

(e) 2

$y' = \frac{c}{2\sqrt{x}}$

At $x = a$, the tangent intersects with $y = c\sqrt{x} \Rightarrow c\sqrt{a} = \frac{3}{2}a + 6$

while $\frac{c}{2\sqrt{a}} = \frac{3}{2}$

Hence, $\frac{c}{\sqrt{a}} = 3 \Rightarrow c = 3\sqrt{a}$

therefore, $3a = \frac{3}{2}a + 6 \Rightarrow \frac{3}{2}a = 6 \Rightarrow a = 4$

& $c = 6$

6. $\lim_{x\to\infty} x \sin\left(\frac{4}{x}\right) =$

(a) 4

(b) 1

(c) 1/4

(d) 0

(e) Does not exist

Let $t = \frac{1}{x}$; $x \to \infty$; $t \to \infty$

$\lim_{x\to\infty} x \cdot \sin\left(\frac{4}{x}\right) = \lim_{t\to 0} \frac{\sin(4t)}{t} = 4.$

7. If $h(x) = \dfrac{x \sin x}{g(x)}$; with $g\left(\dfrac{\pi}{2}\right) = 1$ and $h'\left(\dfrac{\pi}{2}\right) = -2$, then $g'\left(\dfrac{\pi}{2}\right)$ is :

(a) $6/\pi$

(b) $-6/\pi$

(c) 0

(d) $4/\pi$

(e) $-4/\pi$

$g(x) = x\sin x / h(x)$

$g'(x) = \dfrac{(\sin x + x\cos x)h(x) - h'(x)x\sin x}{h^2(x)}$

$g(\frac{\pi}{2}) = 1 \Rightarrow h(\frac{\pi}{2}) = \frac{\pi}{2}$.

$g'(\frac{\pi}{2}) = \dfrac{(1+0)(\frac{\pi}{2}) - (-2)\frac{\pi}{2}}{\frac{\pi^2}{4}}$

$= \dfrac{12}{2\pi} = \dfrac{6}{\pi}$

8. The point(s) (x, y) on $y = \dfrac{x}{x-5}$ at which the tangent line(s) is (are) perpendicular to $y = 5x - 4$ is (are):

(a) $(0, 0)$ and $(10, 2)$

(b) $(0, 0)$ only

(c) $(5, 2)$ only

(d) $(10, 2)$ only

(e) $(0, 0)$ and $(5, 2)$

the tangent slope needs to be equal to $-1/5$

$y' = \dfrac{(x-5)-x}{(x-5)^2} = \dfrac{-5}{(x-5)^2} \Rightarrow \dfrac{-5}{(x-5)^2} = \dfrac{-1}{5}$

$\Rightarrow (x-5)^2 = 25$

$\begin{cases} (x-5) = -5 \Rightarrow x = 0 \text{ \& } y = 0 \\ (x-5) = 5 \Rightarrow x = 10 \text{ \& } y = 2. \end{cases}$

9. An equation of the tangent line to the curve $y = \sqrt[3]{x} - x^3$ at $x = 1$ is:

(a) $8x + 3y = 8$

(b) $3x + 8y = 3$

(c) $8x - 3y = 8$

(d) $3x - 8y = 3$

(e) $-3x + 8y = 3$

$y' = \frac{1}{3} x^{-2/3} - 3x^2 \; ; \; y(1) = 0$

at $x = 1 \Rightarrow y' = \frac{1}{3} - 3 = -8/3$

$(T): (y - y_0) = m(x - x_0)$

$y = -\frac{8}{3}(x - 1)$

$\Rightarrow 3y = -8x + 8$

[redacted] $8x + 3y = 8$

10. A particle moves according to the position function $s(t) = t^2 e^{-t}$; $t \geq 0$, where t is in seconds and s is in meters. What is the total distance traveled by the particle during the first 6 seconds?

(a) $8e^{-2} - 36e^{-6}$

(b) $4e^{-2} - 36e^{-6}$

(c) $4e^{-2}$

(d) $36e^{-6}$

(e) $8e^{-2}$

$v(t) = s'(t) = 2t e^{-t} + t^2(-1)e^{-t}$

$= e^{-t} . t(2 - t)$

$v(t) = s'(t) = 0$ iff $t = 0$ or $t = 2$ sec.

$s(0) = 0 \; ; \; s(2) = 4e^{-2} \Rightarrow d_1 = |4e^{-2} - 0| = 4e^{-2}$

$s(6) = 36e^{-6} \Rightarrow d_2 = |36e^{-6} - 4e^{-2}|$

$d_2 = 4e^{-2} - 36e^{-6}$

$\Rightarrow d = d_1 + d_2 = 8e^{-2} - 36e^{-6}$

11. If $F(x) = 3f(4e^x) \cdot \cos x$, and $f'(4) = \dfrac{-1}{2}$ then $F'(0) = ?$

(a) -6

(b) -3

(c) 3

(d) 6

(e) 4

$$F'(x) = 3f'(4e^x)\cdot(4e^x)\cos x + 3f(4e^x)(-\sin x)$$

$$F'(0) = 3f'(4)(4) + \underbrace{3f(4)(-\sin 0)}_{=0}$$

$$F'(0) = -\frac{3}{2}(4) = -6.$$

12. If $y = \tan^{-1}\left(\dfrac{b + a\cos x}{a - b\cos x}\right)$, where a and b are non-zero constants, then $\dfrac{dy}{dx} =$

(a) $\dfrac{-\sin x}{1 + \cos^2 x}$

(b) $\dfrac{ab\sin x}{(a - b\cos x)^2}$

(c) $\dfrac{a^2\cos x}{b^2(1 + \sin^2 x)}$

(d) $\dfrac{a^2 - b^2}{(a - b\cos x)^2}$

(e) 0

$$y' = \frac{1}{1 + \left(\frac{b + a\cos x}{a - b\cos x}\right)^2} \cdot \left(\frac{(-a\sin x)(a - b\cos x) + (b\sin x)(b + a\cos x)}{(a - b\cos x)^2}\right)$$

$$= \frac{-(a^2 + b^2)\sin x}{(a - b\cos x)^2 + (b + a\cos x)^2}$$

$$= \frac{-(a^2 + b^2)\sin x}{a^2 - \cancel{2ab\cos x} + b^2\cos^2 x + b^2 + \cancel{2ab\cos x} + a^2\cos^2 x}$$

$$= \frac{-(a^2 + b^2)\sin x}{(a^2 + b^2) + (a^2 + b^2)\cos^2 x} = \frac{-\sin x}{1 + \cos^2 x}$$

13. If the curves $y = 3 - x^2$ and $y = Ax^3 + B$ intersect at $(1, 2)$ and their tangent lines at that point are perpendicular, then $7A + B =$

(a) 3

(b) 1

(c) 2

(d) 0

(e) 4

At $(1,2)$: $3-1 = A+B \Rightarrow A+B=2$

slope : $y' = -2x$, $y' = 3Ax^2$

at $x=1$; $y' = -2$ & $y' = 3A$

$\Rightarrow 3A = -\frac{1}{-2} = \frac{1}{2}$

$\Rightarrow A = \frac{1}{6}$ & $B = 2 - \frac{1}{6} = \frac{11}{6}$

$\Rightarrow 7A + B = \frac{7}{6} + \frac{11}{6} = \frac{18}{6} = 3.$

14. The function $y = f(x)$ satisfies the equation $xy'' + y' + xy = 0$, for all x. If $f(0) = 1$, then $f''(0) =$

(a) $\frac{-1}{2}$

(b) $\frac{-3}{4}$

(c) 3

(d) 2

(e) $\frac{1}{2}$

$xy''' + y'' + y'' + xy' + y = 0$

$x = 0 \Rightarrow y = 1$

$0 + 2y''(0) + 0 + 1 = 0$

$y''(0) = -\frac{1}{2}.$

15. $\lim_{\theta \to 0} \dfrac{\sin(2\theta) - \cos(5\,\theta) + \tan(3\,\theta) + 1}{\sin(3\,\theta) + \cos(6\,\theta) - \tan(2\theta) - 1} =$

$$\lim_{\theta \to 0} \frac{\frac{\sin(2\theta)}{\theta} + \frac{\tan(3\theta)}{\theta} - \frac{\cos 5\theta - 1}{\theta}}{\frac{\sin(3\theta)}{\theta} + \frac{\cos(6\theta) - 1}{\theta} - \frac{\tan 2\theta}{\theta}} = \frac{2+3}{3-2} = 5$$

(a) 5

(b) 4

(c) 3

(d) 2

(e) Does not exist

16. If $f(0) = 1$ and $f'(0) = 2$, then $(f^{-1})'(1) =$ $\dfrac{1}{f'(f^{-1}(1))} = \dfrac{1}{f'(0)} = \dfrac{1}{2}$

(a) $\dfrac{1}{2}$

(b) $\dfrac{-1}{2}$

(c) $\dfrac{1}{4}$

(d) $\dfrac{-1}{4}$

(e) -1

17. If $f(x) = xe^{-x}$, then $f^{(n)}(0) =$
$(f^{(n)}(x) : n^{th}$ derivative of f at $x)$

(a) $(-1)^{n+1} \cdot n$

(b) $(-1)^n \cdot n$

(c) $(-1)^n \cdot (n+1)$

(d) $(-1)^{n+1} \cdot (n+1)$

(e) $(-1)^n \cdot (2n)$

$f'(x) = e^{-x} - xe^{-x}$; $f'(0) = 1$

$f''(x) = -e^{-x} - f'(x)$; $f''(0) = -2$

$f'''(x) = e^{-x} - f''(x)$; $f'''(0) = 3$

$f^{(4)}(x) = -e^{-x} - f'''(x)$; $f^{(4)}(0) = -4$

$\Rightarrow f^{(n)}(0) = (-1)^{n+1} \cdot n$

18. The equation of the normal line to $y = \sin(\cos x)$ at $\left(\frac{\pi}{2}, 0\right)$

(a) $y = x - \frac{\pi}{2}$

(b) $y = x + \frac{\pi}{2}$

(c) $y = -x + \frac{\pi}{2}$

(d) $y = -x - \frac{\pi}{2}$

(e) $y = \frac{\pi}{2} x + 1$

$y' = \cos(x) \cdot (-\sin x)$

$y'(\pi/2) = \cos(\underbrace{\cos(\pi/2)}_{0}) \cdot \underbrace{(-\sin \pi/2)}_{-1} = -1$

Normal line's slope $= \frac{1}{1}$

$(N): (y-0) = (1)(x - \pi/2)$

$\Rightarrow y = x - \pi/2$

19. If $y = (2x)^{4x}$, then $y' =$

(a) $4y(1 + \ln(2x))$

(b) $4y(x + \ln(2x))$

(c) $(4x) \cdot (2x)^{4x-1}$

(d) $8y \ln(2x)$

(e) $4xy(1 + \ln(4x))$

$\ln y = 4x \cdot \ln(2x)$

$\frac{y'}{y} = 4x \cdot \frac{1}{2x}(2) + \ln(2x) \cdot 4$

$\frac{y'}{y} = 4 + 4\ln(2x)$

$y' = 4(1+\ln(2x)) \cdot y$

20. A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along the two different legs of the triangle. If the area of the triangle formed by the positions of the two people and their starting point (the right angle) is changing at $4\,m^2/s$, then how fast are the two people moving when they are $5m$ from the right angle?

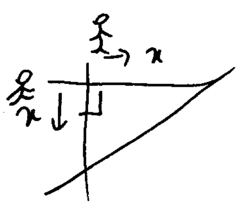
(a) $0.8\,m/s$

(b) $1.6\,m/s$

(c) $0.4\,m/s$

(d) $2.0\,m/s$

(e) $2.4\,m/s$



~~A(t)~~ Area $= \frac{1}{2} x^2(t)$

$\frac{\partial A}{\partial t} = 4 m^2/s = \frac{2}{2} x x'$

$\Rightarrow x' = \frac{4}{x}$

when $x = 5m \rightarrow x' = 4/5 = 0.8\, m/sec$