MATH 101.15 (Term 171)

Quiz 4 (Sects. 3.5, 3.6 & 3.7) Duration: 20min

Name: ID number:

1.)(3pts) Find the slope of the tangent line to: $\tan^{-1}(\frac{xy}{4}) = \frac{\pi}{32}(x^2+y^2)$ at $(2,2)$.

2.)(3pts) Find $f'(0)$ if $f(x) = \frac{(x-2)^4(x^2+3)^3}{(x-1)^2\cos x}$.

3.)(4pts) A particle moves with the position function $s(t) = \sin(\frac{\pi}{2}t) + \cos(\frac{\pi}{2}t)$ (t in seconds and s in meters). Find the total distance traveled by the particle during the time $t \in [0,1]$.

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1) $\tan^{-1}\left(\frac{xy}{4}\right) = \frac{\pi}{32}(x^2+y^2)$

We differentiate with respect to x

$$\frac{1}{4}\left(y + x\frac{dy}{dx}\right)\frac{1}{1+\left(\frac{xy}{4}\right)^2} = \frac{\pi}{32}\left(2x + 2y\frac{dy}{dx}\right)$$

let $m = \left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}}$

$$\frac{1}{4}(2+2m)\frac{1}{1+1} = \frac{\pi}{32}(4+4m)$$

$$\frac{1}{4} + \frac{m}{4} = \frac{\pi}{8} + \frac{\pi}{8}m$$

$$m = \frac{\frac{1}{4} - \pi/8}{\frac{\pi}{8} - \frac{1}{4}} = -1$$

2) $f(x) = \frac{(x-2)^4(x^2+3)^3}{(x-1)^2\cos x}$

$$\ln f(x) = 4\ln|x-2| + 3\ln(x^2+3) - 2\ln|x-1| - \ln\cos x$$

We differentiate wrt x,

$$\frac{f'(x)}{f(x)} = \frac{4}{x-2} + \frac{6x}{x^2+3} - \frac{2}{x-1} + \tan x$$

$$\left.\begin{array}{l}\frac{f'(0)}{f(0)} = -2+2 = 0 \\ f(0) = 2^4 \cdot 3^3 \neq 0\end{array}\right) \Rightarrow f'(0) = 0$$

3) $s(t) = \sin\frac{\pi t}{2} + \cos\frac{\pi}{2}t$

$v(t) = \frac{\pi}{2}\cos\frac{\pi t}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}t$: velocity

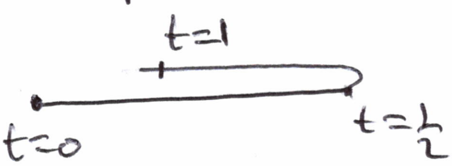
$v(t) = 0 \iff \frac{\pi}{2}\left(\cos\frac{\pi}{2}t - \sin\frac{\pi}{2}t\right) = 0$

$\tan\frac{\pi}{2}t = 1$

$\frac{\pi}{2}t = \pi/4 + k\pi$

$t = \frac{1}{2} + 2k$

t	0		$\frac{1}{2}$		1
$v(t)$		+	0	−	



distance $= |s(\frac{1}{2}) - s(0)| + |s(1) - s(\frac{1}{2})|$

$s(0) = 1, \quad s(\frac{1}{2}) = \sqrt{2}, \quad s(1) = 1$

$\Rightarrow$ distance $= \sqrt{2} - 1 + |1 - \sqrt{2}|$

$= 2(\sqrt{2}-1)$

MATH 101.38 (Term 171)

Quiz 4 (Sects. 3.5, 3.6 & 3.7) Duration: 20min

Name: ID number:

1.)(3pts) Find the slope of the tangent line to: $y = \frac{\cos^{-1} x}{\sin^{-1} x}$ **at** $x = \frac{\sqrt{2}}{2}$.
2.)(3pts) Find $y'(\frac{\pi}{4})$ **if** $y = (2x^{-2} + \ln x)^{\cos 2x}$.
3.)(4pts) A stone is thrown vertically upward with the position function $s(t) = 16t - 4t^2$ **(**t **in seconds and** s **in meters). Find the highest altitude reached by the stone and its velocity when it hits the ground.**

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1) $m = \left.\frac{dy}{dt}\right|_{x=\frac{\sqrt{2}}{2}}$

$$\frac{dy}{dx} = \frac{-\frac{1}{\sqrt{1-x^2}}\sin^{-1}x - \frac{1}{\sqrt{1-x^2}}\cos^{-1}x}{(\sin^{-1}x)^2}$$

$$\left.\frac{dy}{dt}\right|_{x=\frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}\left(\frac{\pi}{4}\right) - \sqrt{2}\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2}$$

$$= -\frac{2\sqrt{2}}{\pi/4} = -\frac{8\sqrt{2}}{\pi}$$

2) $y = (2x^{-2} + \ln x)^{\cos 2x}$

$\Rightarrow \ln y = \cos 2x \ln(2x^{-2} + \ln x)$

we differentiate in x

$$\frac{y'}{y} = -2\sin 2x \ln(2x^{-2} + \ln x) + \frac{(-4x^{-3} + \frac{1}{x})\cos 2x}{2x^{-2} + \ln x}$$

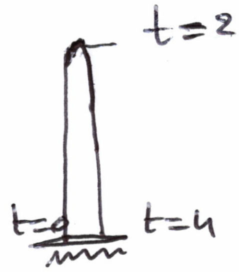
let $x = \pi/4$

$$\frac{y'(\frac{\pi}{4})}{y(\frac{\pi}{4})} = -2\ln\left(\frac{2}{(\frac{\pi}{4})^2} + \ln\frac{\pi}{4}\right)$$

$$y\left(\tfrac{\pi}{4}\right) = \left(\frac{2}{(\frac{\pi}{4})^2} + \ln\frac{\pi}{4}\right)^0 = 1$$

$$y'\left(\tfrac{\pi}{4}\right) = -2\ln\left(\frac{2}{(\frac{\pi}{4})^2} + \ln\frac{\pi}{4}\right)$$

3.)



$s(t) = 16t - 4t^2$

$v(t) = 16 - 8t$: velocity

$v(t) = 0 \Leftrightarrow 16 - 8t = 0$, $t = 2$

a) Highest altitude $= s(2)$
$= 32 - 16 = \underline{16\,m}$

b) At the ground, $s(t) = 0$

$16t - 4t^2 = 0$

$4t(4 - t) = 0$, $t = 0$, $t = 4$

The velocity when it hits the ground is $v(4)$

$v(4) = 16 - 32 = \underline{-16\ m/s}$