MATH 101.15 (Term 171)

Quiz 2 (Sects. 2.5, 2.6 & 2.7) Duration: 20min

Name: ID number:

1.)(3pts) Evaluate $\lim\limits_{z\to -2}\dfrac{\frac{z}{z^2+1}+\frac{2}{5}}{z+2}$.

2.)(4pts)Find all horizontal asymptotes to $f(x)=\begin{cases}\dfrac{\sqrt{x^2-x}}{2x}, & x<0\\ \dfrac{e^x-1}{e^{2x}+1}, & x\ge 0.\end{cases}$

3.)(3pts)Find values of a to make $g(x)=\begin{cases}[[x-2]], & x<1\\ -x^2+ax+3, & x\ge 1\end{cases}$ continuous at $x=1$.

1.) $\lim\limits_{z\to -2}\dfrac{\frac{z}{z^2+1}+\frac{2}{5}}{z+2}=\lim\limits_{z\to -2}\dfrac{(z+2)(2z+1)}{5(z^2+1)(z+2)}$

$=\lim\limits_{z\to -2}\dfrac{2z+1}{5(z^2+1)}$

$=-\dfrac{3}{25}$

2.) $\lim\limits_{x\to +\infty} f(x)=\lim\limits_{x\to +\infty}\dfrac{e^x-1}{e^{2x}+1}$

$=\lim\limits_{x\to +\infty}\dfrac{e^x(1-e^{-x})}{e^{2x}(1+e^{-2x})}$

$=\lim\limits_{x\to +\infty}\dfrac{1-e^{-x}}{e^x(1+e^{-x})}$

$=0$

$\boxed{y=0 \text{ is HA}}$

$\lim\limits_{x\to -\infty} f(x)=\lim\limits_{x\to -\infty}\dfrac{\sqrt{x^2-x}}{2x}$

$=\lim\limits_{x\to -\infty}\dfrac{-x\sqrt{1-\frac{1}{x}}}{2x}$

$=\lim\limits_{x\to -\infty}-\dfrac{\sqrt{1-\frac{1}{x}}}{2}=-\dfrac{1}{2}$

$\boxed{y=-\frac{1}{2} \text{ is a HA}}$

3.) $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)=f(1)$

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}[[x-2]]=-2$

$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(-x^2+ax+3)=a+2$

$\Rightarrow a+2=-2$

$\boxed{a=-4}$

MATH 101.38 (Term 171)

Quiz 2 (Sects. 2.5, 2.6 & 2.7) **Duration: 20min**

Name: **ID number:**

1.)(3pts) Evaluate $\lim_{z\to 2} \dfrac{\frac{z^2}{z+1} - \frac{4}{3}}{z-2}$.

2.)(4pts)Find all horizontal asymptotes to $f(x) = \begin{cases} \dfrac{e^x - 1}{e^{2x}+1}, & x \le 0 \\ \dfrac{\sqrt{x^2+x}}{2x}, & x > 0. \end{cases}$

3.)(3pts)Find values of a **to make** $g(x) = \begin{cases} ax^2 + x - 4, & x \le 1 \\ [[x-1]], & x > 1 \end{cases}$ **continuous at** $x = 1$.

1.) $\lim_{z\to 2} \dfrac{\frac{z^2}{z+1} - \frac{4}{3}}{z-2} = \lim_{z\to 2} \dfrac{3z^2-4z-4}{3(z+1)(z-2)}$

$= \lim_{z\to 2} \dfrac{(3z+2)(z-2)}{3(z+1)(z-2)}$

$= \lim_{z\to 2} \dfrac{3z+2}{3(z+1)} = \dfrac{8}{9}$

2.) $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \dfrac{e^x - 1}{e^{2x}+1} = -1$

$\boxed{y = -1 \text{ is HA}}$

$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \dfrac{\sqrt{x^2+x}}{2x}$

$= \lim_{x\to +\infty} \dfrac{x\sqrt{1+\frac{1}{x}}}{2x}$

$= \lim_{x\to +\infty} \dfrac{\sqrt{1+\frac{1}{x}}}{2}$

$= \dfrac{1}{2}$

$\boxed{y = \frac{1}{2} \text{ is a HA}}$

3.) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (ax^2+x-4)$

$= a-3$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [[x-1]] = 0$

$\Rightarrow a-3=0$

$\boxed{a=3}$