
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
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STAT 319: Probability & Statistics for Engineers & Scientists

Semester 163, Quiz 4

Sunday August 20, 2017

Name:

ID #:

Q.No.1: - It is known that in a chemical process the purity of oxygen is related to the percentage hydrocarbons that are present in the main condenser. The purity of the oxygen in % was measured in 20 hydrocarbon levels and the following regression summary information were obtained: $S_{xx} = 0.68$, $S_{xy} = 10.18$, $S_{yy} = 173.37$,

where the fitted equation is $\hat{Y} = 75.20 + 14.97X$. Based on this,

a) Compute and interpret the correlation coefficient

b) Compute error variance.

c) Test significance of the regression slope with a significance level of 1%. Use p-value approach.

Q.No.2: - Consider the following MINITAB output:

The regression equation is
 $y = 254 + 2.77 x_1 - 3.58 x_2$

Predictor	Coef	SE Coef	T	P
Constant	253.810	4.718	?	?
x1	2.7738	0.1846	15.02	?
x2	-3.5753	0.1526	?	?

S = 5.05756 R-Sq = ? R-Sq(adj) = 98.4%

Analysis of Variance

Source	DF	SS	MS	F
Regression	2	22784	11392	?
Residual Error	?	?	?	
Total	14	23091		

(a) Fill in the missing quantities. You may use the bound for the p-value.

(b) What conclusions can you draw about the significance of regression? The tabulated value from F table is $F_{0.05,2,12} = 3.88$.

(c) What conclusions can you draw about the contribution of the individual predictors to the model?

Q.No.3: - Consider the following partial output from a multiple regression problem:

$$(\mathbf{X}'\mathbf{X}) = \begin{pmatrix} 7 & 51 & 32 \\ 51 & 471 & 235 \\ 32 & 235 & 163.84 \end{pmatrix}, \quad SS_E = 27.58 \quad \text{and} \quad (\mathbf{X}'\mathbf{X})^{-1} = \begin{pmatrix} 1.7996 & -0.06854 & -0.25316 \\ -0.06854 & 0.01008 & -0.00107 \\ -0.25316 & -0.00107 & 0.05708 \end{pmatrix}$$

Use the above to answer the following two problems:

(a) Estimate error variance.

(b) Test the significance of β_1 at 5% level of significance. The estimated value for β_1 is 1.4974.

$$S_{XX} = \sum X^2 - \frac{(\sum X)^2}{n}, \quad S_{YY} = \sum Y^2 - \frac{(\sum Y)^2}{n}, \quad S_{XY} = \sum XY - \frac{\sum X \sum Y}{n}$$

$$Y = \beta_0 + \beta_1 X + \varepsilon, \quad \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X, \quad e = Y - \hat{Y}, \quad \hat{\beta}_1 = \frac{S_{XY}}{S_{XX}}, \quad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$SS_T = SS_R + SS_E, \quad SS_T = S_{YY}, \quad SS_R = \hat{\beta}_1 S_{XY}, \quad SS_E = SS_T - SS_R, \quad \hat{\sigma}^2 = \frac{SS_E}{n-2}$$

$$r_{XY} = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}} = \hat{\beta}_1 \sqrt{\frac{S_{XX}}{S_{YY}}} \quad \text{and} \quad R^2 = \frac{SS_R}{SS_T}$$

$\hat{\beta}_0 \pm T_{\frac{\alpha}{2}, v} \sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}} \right]}$	$T = \frac{\hat{\beta}_0 - \beta_{00}}{\sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}} \right]}}, v = n - 2$
$\hat{\beta}_1 \pm T_{\frac{\alpha}{2}, v} \sqrt{\frac{\hat{\sigma}^2}{S_{XX}}}$	$T = \frac{\hat{\beta}_1 - \beta_{10}}{\sqrt{\frac{\hat{\sigma}^2}{S_{XX}}}}, v = n - 2$
$\hat{y}_0 \pm T_{\frac{\alpha}{2}, v} \sqrt{\hat{\sigma}^2 \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{XX}} \right]}$	$v = n - 2$
$\hat{\mu}_{Y X=x_0} \pm T_{\frac{\alpha}{2}, v} \sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{XX}} \right]}$	$v = n - 2$

$$y = X\beta + \varepsilon, \quad \hat{Y} = X\hat{\beta}, \quad e = Y - \hat{Y}, \quad \hat{\beta} = (X'X)^{-1}X'Y, \quad s.e.(\hat{\beta}_j) = \sqrt{\hat{\sigma}^2 C_{jj}}$$

$$SS_T = SS_R + SS_E, \quad SS_T = Y'Y - \frac{(\sum Y)^2}{n}, \quad Y'Y = \sum Y^2, \quad SS_R = \hat{\beta}'X'Y - \frac{(\sum Y)^2}{n},$$

$$SS_E = SS_T - SS_R, \quad \hat{\sigma}^2 = \frac{SS_E}{n-p}, \quad R^2 = \frac{SS_R}{SS_T} \quad \text{and} \quad R_{adj}^2 = 1 - \frac{SS_E / (n-p)}{SS_T / (n-1)}$$

$$F = \frac{SS_R / k}{SS_E / (n-p)}, \quad v_1 = k \quad \text{and} \quad v_2 = n-p$$

$$T = \frac{\hat{\beta}_j - \beta_{j0}}{s.e.(\hat{\beta}_j)}, \quad v = n-p \quad \hat{\beta}_j \pm T_{\frac{\alpha}{2}, v} s.e.(\hat{\beta}_j)$$

$$\hat{y}_0 \pm T_{\frac{\alpha}{2}, v} \sqrt{\hat{\sigma}^2 \left[1 + x_0'(X'X)^{-1}x_0 \right]}, \quad v = n-p$$

$$\hat{\mu}_{Y|X=x_0} \pm T_{\frac{\alpha}{2}, v} \sqrt{\hat{\sigma}^2 \left[x_0'(X'X)^{-1}x_0 \right]}, \quad v = n-p$$