KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS·

Department of Mathematics and Statistics

Math 301 Final Exam (163)

Name:...ID:....................................Sec:........Ser:....

Exercise/Question #	Mark	
1		20
2		20
3		20
4		20
Q1		10
Q2		10
Q3		10
Q4		10
Q5		10
Q6		10
Total		140

PART 1: Written

Exercise #1: (20 pts) Solve the boundary value problem using the Laplace transform

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \ , \ 0 < x < 1 \ , \ t > 0 \\ u(0,t) = 0 \ , \ u(1,t) = 0 \ , \ t \succeq 0 \\ u(x,0) = 0 \ , \ \frac{\partial u}{\partial t}(x,0) = 4\sin(3\pi x) \ , \ 0 < x < 1 \end{cases}$$

Exercise #2: (20 pts) Solve the following problem using the Fourier transform leaving the solution in integral form

$$\begin{cases} \frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \ , \ -\infty < x < \infty \ , \ t > 0 \\ u(x,0) = e^{-3|x|}, \end{cases}$$

where $\lim_{|x|\to\infty} u(x,t) = 0$ and $\lim_{|x|\to\infty} u_x(x,t) = 0$.

Exercise #3: (20 pts) Consider the Sturm-Liouville problem

$$\begin{cases} y'' + y' = \lambda y \quad . \quad , & 0 < x < 1 \\ \quad y(0) = 0 \quad , & y(1) = 0 \end{cases}$$

(a) Write the differential equation in self-adjoint form specifying the weight function as well as an orthogonality relation.
(b) Find the eigenvalues and corresponding eigenfunctions
(c) Use (a) and (b) above to obtain the eigenfunctions expansion of $f(x) = e^{-\frac{x}{2}}$, $0 < x < 1$ as well as the value of the series at $x = \frac{1}{2}$.

Exercise #4: (20 pts)
(a) (10 pts) Find the first four terms of the Fourier Legendre series for the function

$$f(x) = \begin{cases} -1\ , & -1 < x < 0 \\ 2\ , & 0 < x < 1 \end{cases}$$

.**(b)** (10 pts) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 0\ , & x < -2 \\ -1\ , & -2 < x < 0 \\ -2\ , & 0 < x < 2 \\ 0\ , & x > 2 \end{cases}$$

Part 2: MCQ

Q1) (10 pts) Consider the equation $y(t) = \cos t + \int_0^t e^{-\tau} y(t-\tau)d\tau$ and let $Y(s)$ denote the Laplace transform of $y(t)$ then

(A) $Y(s) = \frac{s^2-1}{s^2+1}$

(B) $Y(s) = \frac{1}{s^2+1}$

(C) $Y(s) = \frac{s+2}{s^2-1}$

(D) $Y(s) = \frac{s+1}{s^2+1}$

(E) $Y(s) = \frac{s+2}{s^2+1}$

Q2) (10 pts) The inverse Laplace transform of

$$F(s) = \frac{2s^2 - s + 1}{s^2 + s + 1}$$

is

(A) $2\delta(t) - 3e^{-\frac{1}{2}t}\left[\cos\left(\frac{1}{2}t\right) - \frac{1}{9}\sqrt{3}\sin\left(\frac{1}{2}t\right)\right]$

(B) $2u(t) - 3e^{-\frac{1}{2}t}\left[\cos\left(\sqrt{3}t\right) - \frac{1}{9}\sqrt{3}\sin\left(\sqrt{3}t\right)\right]$

(C) $\delta(t) - 3e^{-t}\left[\cos\left(\frac{1}{2}\sqrt{3}t\right) - \frac{1}{9}\sqrt{3}\sin\left(\frac{1}{2}\sqrt{3}t\right)\right]$

(D) $2\delta(t) - 2e^{-\frac{1}{2}t}\left[\cos\left(\sqrt{3}t\right) - \frac{1}{9}\sqrt{3}\sin\left(\sqrt{3}t\right)\right]$

(E) $2\delta(t) - 3e^{-\frac{1}{2}t}\left[\cos\left(\frac{1}{2}\sqrt{3}t\right) - \frac{1}{3\sqrt{3}}\sin\left(\frac{1}{2}\sqrt{3}t\right)\right]$

Q3) (10 pts) The Laplace transform of $f(t) = te^{-2t}\sin(3t)$, .is

(A) $\frac{6s+12}{(s^2+4s+13)^2}$
(B) $\frac{6s+1}{(s^2+2s+13)^2}$
(C) $\frac{s+12}{(s^2+4s+1)^2}$
(D) $\frac{6s+12}{(s^2+4s+9)^2}$
(E) $\frac{6s-12}{(s^2-4s+13)^2}$

Q4) (10 pts) The Fourier series of $f(x) = x$, $-\pi < x < \pi$ is

(A) $\sum_{k\succeq 1} \frac{2(-1)^{k+1}}{\pi} \sin(kx)$
(B) $\sum_{k\succeq 1} \frac{(-1)^{k+1}}{\pi} \sin(kx)$
(c) $\sum_{k\succeq 1} \frac{(-1)^{k+1}}{\pi^2} \sin(kx)$
(D) $\sum_{k\succeq 1} \frac{2(-1)^{k+1}}{\pi} \cos(kx)$
(E) $\sum_{k\succeq 1} \frac{2(-1)^{k+1}}{k} \sin(kx)$

Q5) (10 pts) The Fourier-Bessel series expansion of $f(x) = x$, $0 < x < 3$, in Bessel functions of order one that satisfy the boundary condition $J_1(3\alpha) = 0$ is

(A) $2\sum_{n\geq 1} \frac{1}{\alpha_n J_2(2\alpha_n)} J_1(3\alpha_n x)$
(B) $\sum_{n\geq 1} \frac{1}{\alpha_n J_2(3\alpha_n)} J_1(\alpha_n x)$
(C) $2\sum_{n\geq 1} \frac{1}{\alpha_n J_2(\alpha_n)} J_1(\alpha_n x)$
(D) $3\sum_{n\geq 1} \frac{(-11)^n}{\alpha_n J_2(3\alpha_n)} J_1(\alpha_n x)$
(E) $2\sum_{n\geq 1} \frac{1}{\alpha_n J_2(3\alpha_n)} J_1(\alpha_n x)$

Q6) (10 pts) The general solution of $x^2y'' + xy' + (3x^2 - 1)y = 0$, is:

(A) $c_1 J_1(x) + c_2 Y_1(x)$
(B) $c_1 J_1\left(\sqrt{3}x\right) + c_2 Y_1\left(\sqrt{3}x\right)$
(C) $c_1 J_{\sqrt{3}}(x) + c_2 Y_{\sqrt{3}}(x)$
(D) $c_1 J_1\left(\sqrt{3}x\right) + c_2 J_{-1}\left(\sqrt{3}x\right)$
(E) $c_1 J_0\left(\sqrt{3}x\right) + c_2 Y_0\left(\sqrt{3}x\right)$