

King Fahd University of Petroleum and Minerals  
Department of Mathematics and Statistics  
Math301 Major Exam II (Term 163)  
Duration 120 minutes

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Name:.....ID:.....Serial:... Section:..

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- Mobiles and calculators are not allowed
- Write all the steps clearly

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Exercise #	Mark	Max Marks
1		10
2		15
3		15
4		15
5		12
6		15
7		18
Total		100

**Exercise #1** (10 pts) Find the Laplace transform of

$$f(t) = e^{-t} \sin^2(3t)$$

**Exercise #2** (15 pts) Find the inverse Laplace transform of

$$F(s) = \frac{s^2 + s - 1}{2s^2 + 2s + 1}$$

**Exercise #3** (15 pts) Use the Laplace transform to solve the initial value problem

$$y'' + y' - 6y = 3e^{-t}, \quad y(0) = 0, \quad y'(0) = 0$$

**Exercise #4** (15 pts) Use the Laplace transform to solve for  $f$ , the integral equation

$$f(t) = t - \int_0^t f(t - \tau) \sin(\tau) d\tau$$

**Exercise #5** (12 pts) Use the Laplace transform to solve the initial value problem

$$y'' + 2y' + 5y = \delta(t - 2), \quad y(0) = 2, \quad y'(0) = 0$$

**Exercise #6** (15 pts) Are there values of the parameters  $a$  and  $b$  which make the functions

$$f_1(x) = x + a, \quad f_2(x) = x^2 + bx$$

orthonormal on the interval  $[-1, 1]$ , if yes, find them.

**Exercise #7** (18 pts)

a) Find the Fourier series expansion of

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$$

b) Use the result from (a) to show that

$$\frac{\pi^2}{12} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$$