MATH 301 (163) **MAJOR EXAM I** **Wednesday July 26, 2017**

Name:... ID:....................................Section:.....Ser:.....

IMPORTANT: In Exercises 4, 5 and 6, students should clearly state the hypotheses of the corresponding theorem.

Exercise #1: (14pts) Consider the function f from R^3 to R defined by

$$f(x, y, z) = xy + e^{yz}$$

(a) Find the directional derivatives of f at the point $P_0(1, 1, 1)$ in the direction of the point $P_1(2, 1, 0)$.
(b) Find the direction in which the directional derivative has maximum value. What is this maximum value?

Exercise #2: (18pts) Consider the vector field

$$F(x, y, z) = y^2e^z\mathbf{i} + 2xye^z\mathbf{j} + xy^2e^z\mathbf{k}$$

(a) Is the vector field F conservative? if yes, find a potential $\varphi(x, y, z)$.
(b) Find the work done by F along the straight line from $P_0(1, 1, 0)$ to $P_1(1, 1, 1)$.

Exercise #3: (18pts) Find the surface integral

$$I = \int\int_S z dS$$

where the surface S is defined by the portion of the plane $2x + 3y + 4z = 24$,
in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$.
Exercise #4: (22pts) Verify Green's theorem for the vector field $F(x, y) = x\mathbf{i} + xy^2\mathbf{j}$ and C is the curve $x^2 + y^2 = 4$ oriented counterclockwise.

Exercise #5: (14pts) Use Stokes' theorem to evaluate

$$\int\int_S (\text{curl}\, F) \cdot n dS$$

where $F(x, y, z) = y\mathbf{i} + zy^2\mathbf{j} + \cos(z)\mathbf{k}$ and S is the portion of the paraboloid $z = 16 - x^2 - y^2$, $z \geq 0$, oriented upward.

Exercise #6: (14pts) Use the divergence theorem to evaluate

$$\int\int_S F \cdot n dS$$

where $F(x, y, z) = xy^2\mathbf{i} + 2y\mathbf{j} - zy^2\mathbf{k}$ and S is the surface of the solid defined by $x^2 + y^2 + z^2 \leq 4$ and $z \geq 0.$, riented outward.