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Math201.01, Quiz #1, Term 163

Name: Solutions

ID #: **Serial #:**

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1. **[3 points]** Describe and sketch, with directions, the parametric curve given by

$$x = -sint, \qquad y = 1 - cost, \qquad 0 \le t \le 3\pi/2.$$

2. **[3 points]** Find the length of the polar curve $r = \theta^2,\ 0 \le \theta \le \pi$.
3. **[4 points]** Find the area of the polar region that lies **inside** the curve $r = 2cos\theta$ and **outside** the curve $r = 1$.

Good luck,

Ibrahim Al-Rasasi

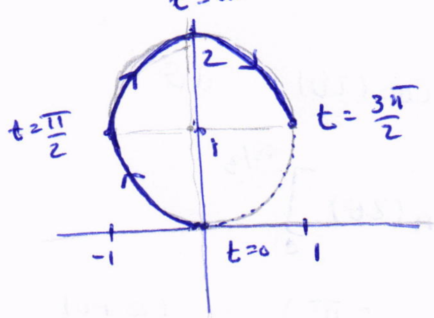
1.5

$\boxed{1}$ • as $\sin^2 t + \cos^2 t = 1$, then $(-x)^2 + (1-y)^2 = 1$, and so $x^2 + (y-1)^2 = 1$, a Circle with center $(0,1)$ and radius 1

• For directions

t	(x,y)
0	$(0,0)$ ← initial pt
$\frac{\pi}{2}$	$(-1,1)$
π	$(0,2)$
$\frac{3\pi}{2}$	$(1,1)$ ← terminal pt

0.5



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the Solid part of the above circle

[2] $r = \theta^2, \quad 0 \le \theta \le \pi$

$$L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad 0.5 \qquad ; \quad \frac{dr}{d\theta} = 2\theta \quad 0.5$$

$$= \int_0^{\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{\theta^2(\theta^2+4)}\, d\theta = \int_0^{\pi} \theta\sqrt{\theta^2+4}\, d\theta \quad 1 \quad : \quad u = \theta^2 + 4 \Rightarrow du = 2\theta\, d\theta$$

$$\theta = 0 \Rightarrow u = 4$$
$$\theta = \pi \Rightarrow u = \pi^2 + 4$$

$$= \frac{1}{2}\int_4^{\pi^2+4} u^{1/2}\, du = \frac{1}{2}\cdot\frac{2}{3}\, u^{3/2}\Big]_4^{\pi^2+4}$$

$$= \frac{1}{3}\cdot\left[(\pi^2+4)^{3/2} - 4^{3/2}\right] = \frac{1}{3}\left[(\pi^2+4)^{3/2} - 8\right] \quad 1$$

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[3] . pts of intersection : $2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3}$ 0.5

. By symmetry about the polar axis :

$$A = 2 \cdot \int_0^{\pi/3} \frac{1}{2}\left[(2\cos\theta)^2 - (1)^2\right] d\theta \quad 1.5$$

$$= \int_0^{\pi/3} 4\cos^2\theta - 1 \quad d\theta$$

$$= \int_0^{\pi/3} 4\cdot\frac{1+\cos(2\theta)}{2} - 1 \quad d\theta \quad 0.5$$

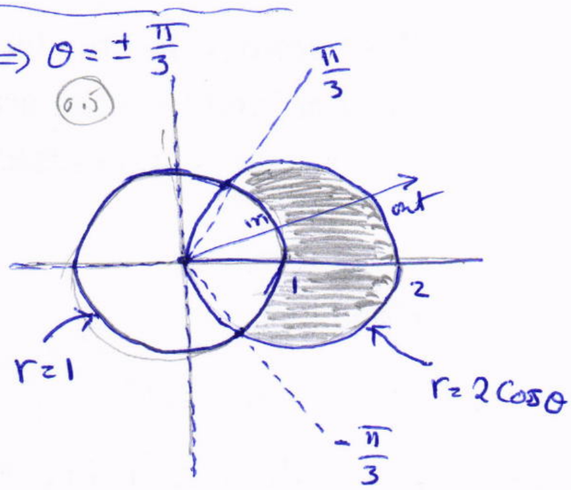
$$= \int_0^{\pi/3} 2 + 2\cos(2\theta) - 1 \quad d\theta$$

$$= \int_0^{\pi/3} 1 + 2\cos(2\theta) \quad d\theta$$

$$= \theta + \sin(2\theta)\Big]_0^{\pi/3}$$

$$= \frac{\pi}{3} + \sin\left(2\frac{\pi}{3}\right) - (0+0)$$

$$= \frac{\pi}{3} + \frac{\sqrt{3}}{2} \quad 0.5$$



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Math201.02, Quiz #1, Term 163

Name:

ID #: Solutions **Serial #:**

1. **[3 points]** Describe and sketch, with directions, the parametric curve given by

$$x = t^2, \qquad y = lnt\,, \qquad 0 < t < \infty.$$

2. **[3 points]** Find the area of the surface generated by rotating the parametric curve $x = 3t - t^3$, $y = 3t^2$, $0 \le t \le 1$ about the $x-axis$.
3. **[4 points]** Find the area of the polar region that lies **inside** the curve $r = 1 - sin\theta$ and **outside** the curve $r = 1$.

Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ $x = t^2 \Rightarrow |t| = \sqrt{x} \Rightarrow t = \sqrt{x}$, since $t > 0$

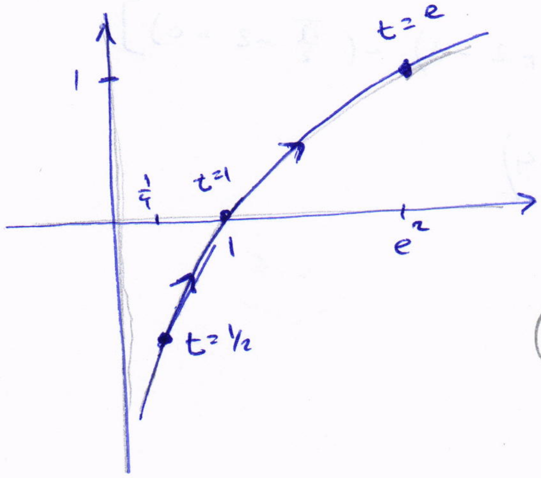
$\Rightarrow y = \ln\sqrt{x}$

1.5 $\Rightarrow y = \frac{1}{2}\ln x$, a logarithmic curve

For directions

t	(x,y)
$\frac{1}{2}$	$(\frac{1}{4}, -\ln 2)$
1	$(1,0)$
e	$(e^2, 1)$

0.5



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[2] $x = 3t - t^3,\ y = 3t^2,\ 0 \le t \le 1$

$$S = \int_0^1 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \quad 0.5$$

$\frac{dx}{dt} = 3 - 3t^2,\quad \frac{dy}{dt} = 6t$ 0.5

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (9 - 18t^2 + 9t^4) + 36t^2$$
$$= 9 + 18t^2 + 9t^4$$
$$= (3 + 3t^2)^2$$
$$\sqrt{\quad} = 3 + 3t^2$$

$$= 2\pi \int_0^1 3t^2 \cdot (3 + 3t^2)\, dt$$
$$= 18\pi \int_0^1 (t^2 + t^4)\, dt$$
$$= 18\pi \cdot \left.\frac{t^3}{3} + \frac{t^5}{5}\right]_0^1 \quad 0.5$$
$$= 18\pi \cdot \left(\frac{1}{3} + \frac{1}{5}\right) = 18\pi \cdot \frac{8}{15} = 6\pi \cdot \frac{8}{5} = \frac{48}{5}\pi \quad 0.5$$

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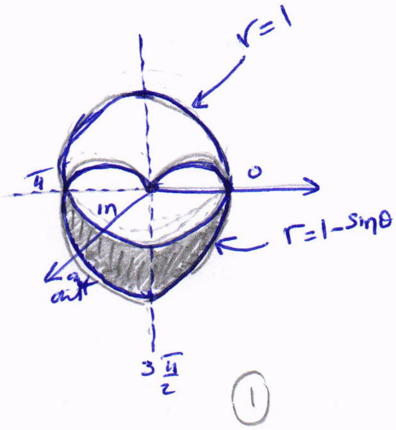
[3] • pts of intersection: $1 - \sin\theta = 1 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi$ 0.5



$$A = \int_\pi^{2\pi} \frac{1}{2}\left[(1 - \sin\theta)^2 - (1)^2\right] d\theta \quad 1.5$$
$$= \int_\pi^{2\pi} \frac{1}{2}\left[1 - 2\sin\theta + \sin^2\theta - 1\right] d\theta$$
$$= \frac{1}{2}\int_\pi^{2\pi} -2\sin\theta + \frac{1}{2}(1 - \cos(2\theta))\, d\theta \quad 0.5$$
$$= \frac{1}{2}\int_\pi^{2\pi} \frac{1}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)\, d\theta$$
$$= \frac{1}{2}\left[\frac{1}{2}\theta + 2\cos\theta - \frac{1}{4}\sin(2\theta)\right]_\pi^{2\pi}$$
$$= \frac{1}{2}\left[(\pi + 2 - 0) - \left(\frac{\pi}{2} - 2 - 0\right)\right]$$
$$= \frac{1}{2}\left(\frac{\pi}{2} + 4\right)$$
$$= \frac{\pi}{4} + 2 \quad 0.5$$