Math201.01, Quiz #2, Term 163

Name:

ID #: **Serial #:**

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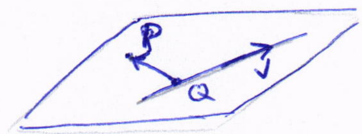
1. [4 points] Find an equation for the plane containing the point $P(3,5,-4)$ and the line $x = 4 - t$, $y = -1 + 2t$, $z = -3t$, $t \in (-\infty, \infty)$.



0.5 • a point on the line is $Q(4,-1,0)$ (when $t=0$)

0.5 • a vector parallel to the line is $\vec{v} = \langle -1, 2, -3 \rangle$

0.5 • $\vec{QP} = \langle -1, 6, -4 \rangle$

• a normal to the plane is

$$\vec{n} = \vec{QP} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & 6 & -4 \\ -1 & 2 & -3 \end{vmatrix} = -10\vec{i} + \vec{j} + 4\vec{k}$$

1 0.5

• an equation for the plane:

0.5
$$-10(x-3) + 1(y-5) + 4(z+4) = 0$$
$$-10x + 30 + y - 5 + 4z + 16 = 0$$
$$-10x + y + 4z + 41 = 0$$

0.5 or $10x - y - 4z = 41$

2. [4 points] Give the name, vertex, axis, and sketch the graph of the equation $x^2 + 3y^2 - 12y - z + 13 = 0$.

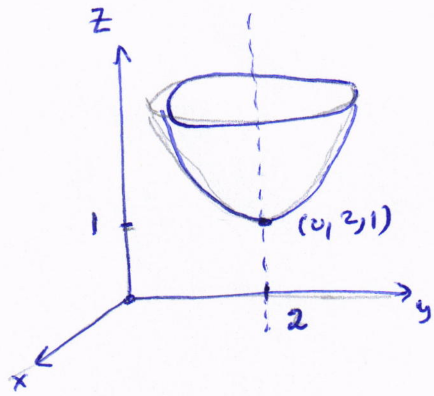
$$z = x^2 + 3y^2 - 12y + 13$$
$$z = x^2 + 3(y^2 - 4y + 4) + 13 - 12$$
$$= x^2 + 3(y-2)^2 + 1$$

1

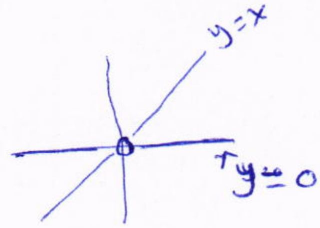
name : an elliptic paraboloid 1

vertex : $(0, 2, 1)$ 0.5

axis : The line parallel to the z-axis & passing through $(0,2,1)$ 0.5



1



3. [3 points] Find the limit: $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$.

Try paths

1 · along the x-axis : y=0

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{xy+y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \left.\frac{xy+y^3}{x^2+y^2}\right|_{y=0} = \lim_{(x,y)\to(0,0)} \frac{0+0}{x^2+0} = \lim_{(x,y)\to(0,0)} 0 = 0$$

1 · along the line y=x :

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{path } y=x}} \frac{xy+y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \left.\frac{xy+y^3}{x^2+y^2}\right|_{y=x} = \lim_{(x,y)\to(0,0)} \frac{x^2+x^3}{x^2+x^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{1}{2}+\frac{1}{2}x = \frac{1}{2}$$

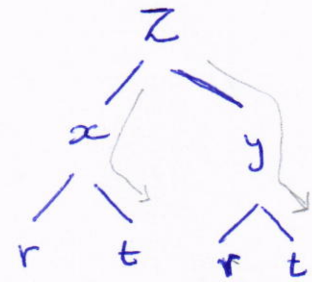
· Since the limits along the two paths are not equal, 0.5

then $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$ DNE. 0.5

4. [4 points] Let $z = tan^{-1}(x^2+y^2),\ x = rlnt,\ y = te^{2r}$. Find $\frac{\partial z}{\partial t}$ when $(r,t) = (1,1)$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} \quad 1$$

$$= \frac{2x}{1+(x^2+y^2)^2}\cdot\frac{r}{t} + \frac{2y}{1+(x^2+y^2)^2}\cdot e^{2r} \quad 2$$



$$\left.\frac{\partial z}{\partial t}\right|_{\substack{r=1\\ t=1}} = 0\cdot 1 + \frac{2e^2}{1+e^8}\cdot e^2$$

$$= \frac{2e^4}{1+e^8} \quad 0.5$$

r=1, t=1

$\Rightarrow x=0,\ y=e^2$ 0.5

Math201.02, Quiz #2, Term 163

Name:

ID #: **Serial #:**

1. [4 points] Give the name, vertex(ces), axis, and sketch the graph of the equation $z^2 - 2 = x^2 + 2y^2 - 2x$.

$z^2 - 2 = x^2 - 2x + 1 + 2y^2 - 1$

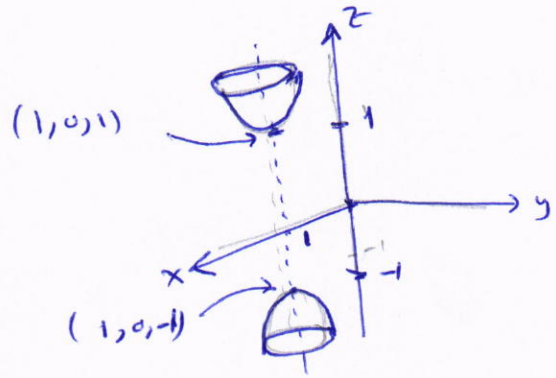
$= (x-1)^2 + 2y^2 - 1$

$-(x-1)^2 - 2y^2 + z^2 = 1$ 1

name: a hyperboloid of 2 sheets 1

verteces : $(1,0,1)$, $(1,0,-1)$ 0.5

axis : the line parallel to the z-axis passing through $(1,0,1)$ & $(1,0,-1)$ 0.5



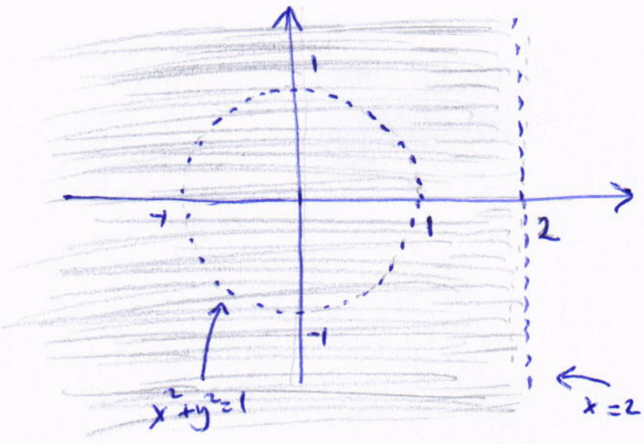
1

2. [4 points] Find and sketch the domain of $f(x,y) = \frac{\ln(2-x)}{1-x^2-y^2}$.

Domain $= \{(x,y) : 2 - x > 0$ and $1 - x^2 - y^2 \neq 0\}$

$= \{(x,y) : x < 2$ and $x^2 + y^2 \neq 1\}$ 1 + 1

= all points to the left (& not on) of the line $x = 2$ excluding the points on the unit circle $x^2+y^2=1$.



2

3. [3 points] Find the limit: $\lim_{(x,y)\to(2,1)} \frac{x^2y^2-xy-2}{\sqrt{2xy}-2}$.

$$= \lim_{(x,y)\to(2,1)} \frac{x^2y^2-xy-2}{\sqrt{2xy}-2} \cdot \frac{\sqrt{2xy}+2}{\sqrt{2xy}+2} \quad 0.5$$

$$= \lim \frac{(x^2y^2-xy-2)(\sqrt{2xy}+2)}{2xy-4} \quad 0.5$$

$$= \lim \frac{(xy-2)(xy+1)}{2(xy-2)} \cdot (\sqrt{2xy}+2) \quad 1$$

$$= \lim_{(x,y)\to(2,1)} \frac{1}{2}(xy+1)(\sqrt{2xy}+2) \quad 0.5$$

$$= \frac{1}{2}(2+1)(2+2) = 6 \quad 0.5$$

4. [4 points] Let $z = tan^{-1}(x/y)$. Find $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y}$ and z_{xx}.

- $\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} = \frac{y^2}{y^2+x^2} \cdot \frac{1}{y} = \frac{y}{x^2+y^2}$ 1
- $\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{-x}{y^2} = \frac{y^2}{y^2+x^2} \cdot \frac{-x}{y^2} = \frac{-x}{x^2+y^2}$ 1
- $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = y \cdot \frac{y}{x^2+y^2} - x \cdot \frac{-x}{x^2+y^2}$

$$= \frac{y^2}{x^2+y^2} + \frac{x^2}{x^2+y^2} = \frac{y^2+x^2}{x^2+y^2} = 1 \quad 1$$

- $z_{xx} = \frac{\partial^2 z}{\partial x^2} = \frac{(x^2+y^2)\cdot 0 - y(2x)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$. 1