King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 201 Major Exam II
The Third Semester of 2016-2017 (163)
Time Allowed: 120 Minutes

Name: Key ID#: ______

Section/Instructor: ______ Serial #: ______

- Mobiles, calculators and smart devices are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		10
2		8
3		14
4		12
5		14
6		14
7		14
8		14
Total		100

Q:1 (10 points) Find an equation of the plane passing through the line of intersection of the planes $x+y+z=1$, $2x-y+2z=2$ and contains the point $P(1,2,3)$.

Sol: Let's find two points on the line $\begin{cases} x+y+z=1 \\ 2x-y+2z=2 \end{cases}$

Setting $x=0$; $\begin{matrix} y+z=1 \\ -y+2z=2 \end{matrix}$ $\Rightarrow$ $z=1, y=0$
$Q(0,0,1)$

Setting $z=0$; $\begin{matrix} x+y=1 \\ 2x-y=2 \end{matrix}$ $\Rightarrow$ $x=1, y=0$
$R(1,0,0)$

$\overrightarrow{PQ} = \langle -1,-2,-2 \rangle$
$\overrightarrow{PR} = \langle 0,-2,-3 \rangle$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -2 \\ 0 & -2 & -3 \end{vmatrix} = \langle 6-4, -3, 2 \rangle$$
$$= \langle 2,-3,2 \rangle$$

Equation of the plane is

$$2(x-1) - 3(y-2) + 2(z-3) = 0$$
$$2x - 3y + 2z = 2-6+6 = 2.$$

Q:2 (a) (04 points) Find the point of intersection of the plane $2x + y - 3z = 7$ and the line $x = 1 - 2t, \quad y = 2 + t, \quad z = 1 + t.$

Sol.

$$2(1-2t) + (2+t) - 3(1+t) = 7$$

$$2 - 4t + 2 + t - 3 - 3t = 7$$

$$-6t = 6$$

$$t = -1$$

Point $(1+2,\ 2-1,\ 1-1) = (3, 1, 0)$

(b) (04 points) Find the distance between the parallel planes $x+2y-z = 4$ and $2x+4y-2z = 3$.

Point on the plane $x + 2y - z = 4$ is $(4, 0, 0)$

Distance between point $(4, 0, 0)$ and plane is

$$\left| \frac{2 \times 4 + 4 \times 0 - 2 \times 0 - 3}{\sqrt{4 + 16 + 4}} \right|$$

$$= \frac{5}{\sqrt{24}} = \frac{5}{2\sqrt{6}}$$

Q:3 (14 points) Consider the surface $x^2 + z^2 - 2x + 6z - y + 10 = 0$.

(a) Reduce the equation to one of the standard forms.

(b) Classify the surface (name, vertex, axis).

(c) Find the traces on the three coordinates planes.

(d) Sketch the surface.

(a) $x^2 - 2x + z^2 + 6z - y = -10$

$(x-1)^2 + (z+3)^2 - y = -10 + 1 + 9$

$(x-1)^2 + (z+3)^2 = y$

(b) It is a paraboloid with vertex $(1,0,-3)$ and axis the line parallel to y-axis. passing through the vertex.

(c) On the xy-plane $(z=0)$:

$y = (x-1)^2 + 9$ (parabala)

On the yz-plane $(x=0)$:

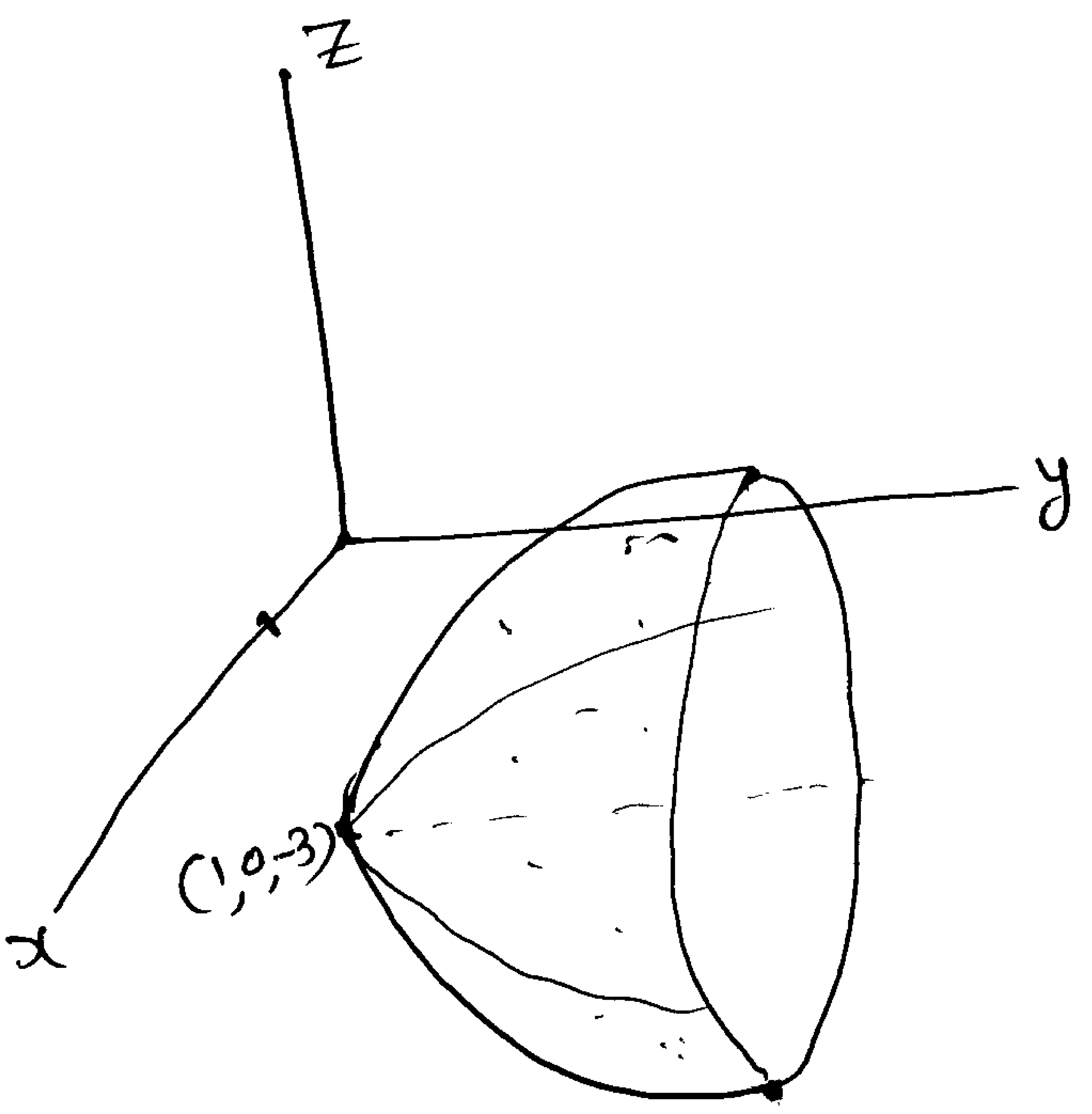
$y = (z+3)^2 + 1$ (parabala)

On the xz-plane $(y=0)$:

$(x-1)^2 + (z+3)^2 = 0$

It is point $(1,0,-3)$

(d)



Q:4 Let $f(x, y) = \sqrt{x^2 - y^2} + \sqrt{1 - x^2}$.

(a) (08 points) Find and sketch the domain of f.

$D = \{(x, y): x^2 - y^2 \geq 0 \text{ and } 1 - x^2 \geq 0\}$

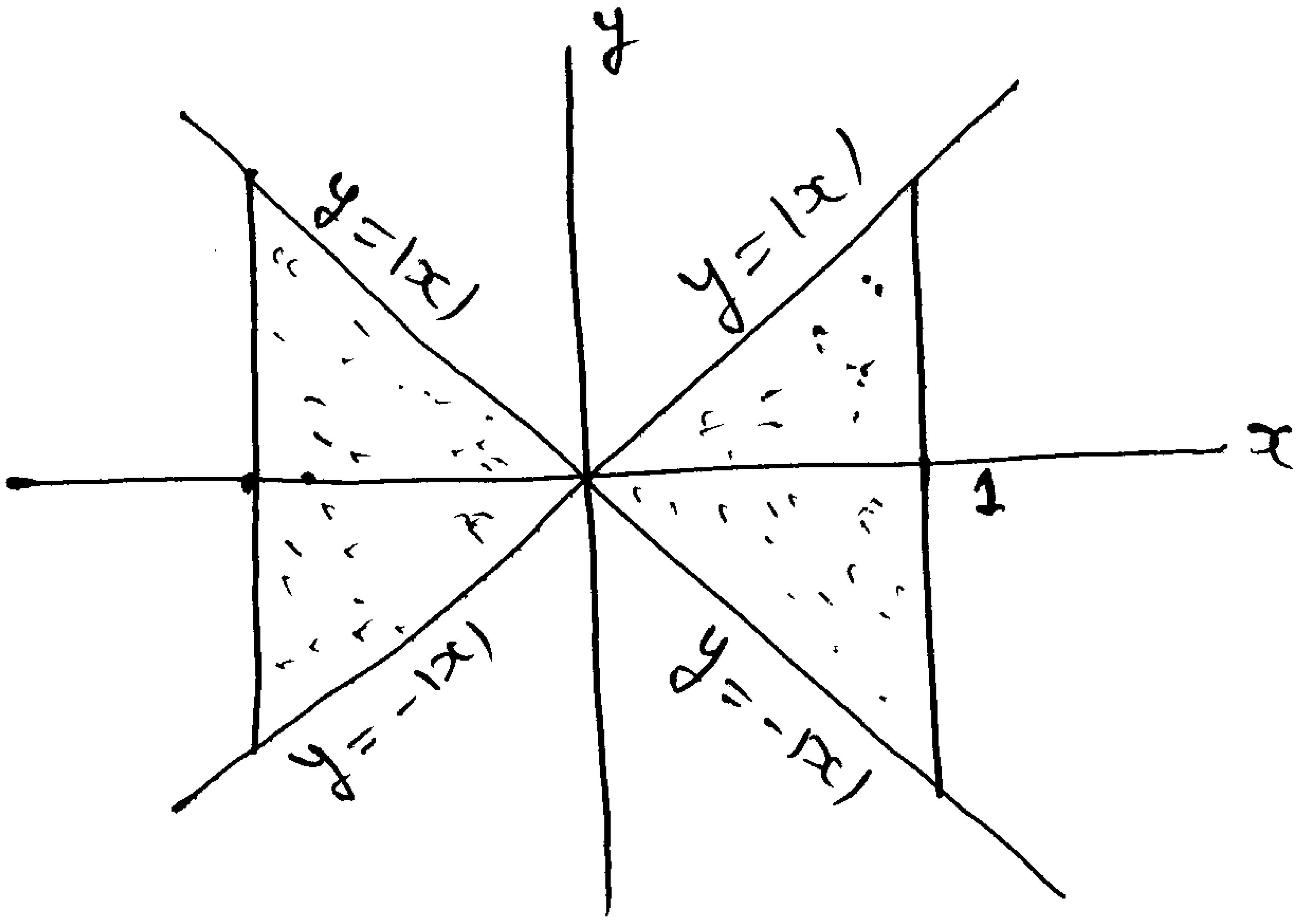
$x^2 - y^2 \geq 0 \Leftrightarrow y^2 \leq x^2 \Leftrightarrow |y| \leq |x|$

$\Leftrightarrow -|x| \leq y \leq |x|$

and $1 - x^2 \geq 0$

$\Leftrightarrow x^2 \leq 1$

$\Leftrightarrow -1 \leq x \leq 1$



(b) (04 points) Find an equation for the level curve of the function $f(x, y)$ through the point $(1, 0)$.

$C = \sqrt{x^2 - y^2} + \sqrt{1 - x^2}$

$C = \sqrt{1 - 0} + \sqrt{1 - 1}$

$C = 1$

Equation of the level curve is

$\sqrt{x^2 - y^2} + \sqrt{1 - x^2} = 1$

(Q:5) (a) (10 points) Find the limit, if it exists,

(i) $$\lim_{(x,y)\to(1,2)} \frac{\frac{1}{x}-\frac{y}{2}}{xy-2}.$$

$$= \lim_{(x,y)\to(1,2)} \frac{-(xy-2)}{2x(xy-2)}$$

$$= \lim_{(x,y)\to(1,2)} -\frac{1}{2x}$$

$$= -\frac{1}{2}$$

(ii) $$\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^3+3y^3}.$$

Along $y=0$: $\lim_{x\to 0} \frac{0}{x^3+0} = 0.$

or $x=0$

Along $y=x$: $\lim_{x\to 0} \frac{2x\,x^2}{x^3+3x^3}$

$$= \lim_{x\to 0} \frac{2x^3}{4x^3} = \frac{1}{2}$$

Limits are not same.

$\Rightarrow$ DNE

(b) (04 points) Determine the set of points at which the function $g(x,y,z) = \dfrac{1}{4-\sqrt{x^2+y^2+z^2-9}}$ is continous.

$D = \{(x,y,z) : x^2+y^2+z^2 \geq 9 \text{ and } x^2+y^2+z^2 \neq 25\}$

Q:6 (a) (06 points) If $s(u, v, w) = u \tan^{-1}(v\sqrt{w})$, find $\dfrac{\partial^2 s}{\partial u \partial w}(0, 1, 2)$

$$s = u \tan^{-1}(v\sqrt{w})$$

$$\frac{\partial s}{\partial u} = \tan^{-1}(v\sqrt{w})$$

$$\frac{\partial^2 s}{\partial u \partial w} = \frac{1}{1+(v\sqrt{w})^2} \cdot \frac{\partial}{\partial w}(v\sqrt{w})$$

$$= \frac{1}{1+v^2 w} \cdot v \frac{1}{2\sqrt{w}}$$

$$\frac{\partial^2 s}{\partial u \partial w}(0,1,2) = \frac{1}{1+2} \cdot \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{6\sqrt{2}}$$

(b) (08 points) Let $u = pq + \ln$ r,, where

$$p = \frac{y^2}{x}, \quad q = y + z \text{ and } r = xz.$$

Use the **Chain Rule** to find the values of $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ at $(x, y, z) = (1, 1, -2)$.

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x}$$

$$= (q)\left(-\frac{y^2}{x^2}\right) + p \cdot 0 + \frac{1}{r} \cdot z$$

$$= (-1)(-1) + \left(-\frac{1}{2}\right)(-2)$$

$$= 1 + 1 = 2$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial y} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial y} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial y}$$

$$= (q)\left(\frac{2y}{x}\right) + (p)(1) + \frac{1}{r} \cdot 0$$

$$= (-1)(2) + (1)(1)$$

$$= -1$$

Q:7 (a) (06 points) Find an equation of the tangent plane to the surface

$$f(x,y) = x^2 - 3xy + \frac{1}{2}y^2 + 3$$

at the point $(2,2,-3)$.

$f_x = 2x - 3y$, $\quad f_x(2,2) = -2$

$f_y = -3x + y$, $\quad f_y(2,2) = -4$

Equation of the tangent plane is

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$z + 3 = (-2)(x-2) + (-4)(y-2)$$

$$z = -2x + 4 - 4y + 8 - 3$$

$$z = -2x - 4y + 9$$

(b) (08 points) Find the linearization of the function $g(x,y) = e^{2y-x}$ at $(2,1)$ and use it to approximate $g(1.95, 1.05)$.

$g_x = -e^{2y-x}$, $\quad g_x(2,1) = -e^0 = -1$

$g_y = 2e^{2y-x}$, $\quad g_y(2,1) = 2$

$g(2,1) = 1$

$$L(x,y) = -1(x-2) + 2(y-1) + 1$$

$$= -x + 2y + 1$$

$$g(1.95, 1.05) \simeq -1.95 + 2.1 + 1$$

$$= 1.15$$

Q:8 a (08 points) Let $f(x,y) = x^2 - xy + y^2 - y$. Find the unit vector **u** such that $D_{\mathbf{u}}f(1,-1) = 0$.

$$\nabla f = \langle 2x - y, -x + 2y - 1 \rangle$$

$$\nabla f(1,-1) = \langle 3, -4 \rangle.$$

Let $\hat{u} = \langle a, b \rangle$. Then

$$a^2 + b^2 = 1 \qquad (*)$$

$$D_{\hat{u}} f(1,-1) = \nabla f(1,-1) \cdot \hat{u}$$

$$0 = 3a - 4b \qquad (**)$$

$$\Rightarrow a = \frac{4}{3} b$$

$(*)$ gives $\frac{16}{9} b^2 + b^2 = 1 \Rightarrow 25 b^2 = 9$

$$\Rightarrow b = \pm \frac{3}{5}$$

$$a = \pm \frac{4}{5}$$

$$\hat{u} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle \text{ or } \left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle.$$

(b) (06 points) Find the **parametric equations** for the normal line to the surface $xyz^2 = 6$ at the point $(3,2,1)$.

$$F(x,y,z) = xyz^2 - 6$$

$$\nabla F = \langle yz^2, xz^2, 2xyz \rangle$$

$$\nabla F(3,2,1) = \langle 2, 3, 12 \rangle$$

Normal line has direction $\langle 2, 3, 12 \rangle$.

The parametric equations for the normal line are

$$x = 3 + 2t$$

$$y = 2 + 3t$$

$$z = 1 + 12t$$