King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 131 (Term 163)

Final Exam – CODE 001

(Duration: 150 minutes. Number of Exercises: 25)

Student Name_____ **Student ID:** _____

Exercise 1
The demand function for a manufacturer's product is $p = 200 - 5q$, where p is the price (in SR) per unit when q units are demanded (per day). Find the maximum revenue that the manufacturer can receive.

Answer: (a) 2000 SR **(b)** 3000 SR **(c)** 4000 SR **(d)** 5000 SR **(e)** 6000 SR

Exercise 2
A manufacturer of a children's toy will break even at a total revenue of 200,000 SR. Fixed costs are 40,000 SR, and each unit of output sells for 5 SR. Determine the variable cost per unit.

Answer: (a) 1 SR **(b)** 2 SR **(c)** 3 SR **(d)** 4 SR **(e)** 5 SR

Exercise 3
Let $= \begin{pmatrix} 2 & 3 & 2 & 6 \\ 0 & 1 & 2 & 1 \\ 3 & 0 & -3 & 6 \end{pmatrix}$. Find the reduced form for the matrix A.

Answer: (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Exercise 4
Let the function $Z = 20x + 30y$ subject to $2x + y \leq 10;\ 3x + 4y \leq 24;\ 8x + 7y \geq 56;\ \ x, y \geq 0$.
Then:

(a) Z has a maximum value at $(\frac{16}{5}, \frac{18}{5})$
(b) Z has a maximum value at $(\frac{7}{3}, \frac{16}{3})$
(c) Z has no maximum value
(d) Z has a minimum value at (7, 0)
(e) Z has a minimum value at (0, 6)

Exercise 5

A produce grower is purchasing fertilizer containing three nutrients: A, B, and C. The minimum weekly requirements are 100 units of A, 200 of B, and 300 of C. There are two popular blends of fertilizer on the market. Blend I, costing 10 SR a bag, contains 2 units of A, 6 of B, and 4 of C. Blend II, costing 8 SR a bag, contains 2 units of A, 2 of B, and 12 of C. How many bags of each blend should the grower buy each week to minimize the cost of meeting the nutrient requirements? Let x = Number of bags of Blend I and y = Number of bags of Blend II. The linear programming problem to minimize cost Z is:

(a) Minimize $Z = 10x + 8y$ subject to $2x + 2y \leq 100$; $6x + 2y \leq 200$; $4x + 12y \leq 300$
(b) Maximize $Z = 8x + 10y$ subject to $2x + 2y \leq 100$; $2x + 6y \leq 200$; $12x + 4y \leq 300$
(c) Minimize $Z = 10x + 8y$ subject to $x + y \geq 50$; $3x + y \geq 100$; $x + 3y \geq 75$
(d) Maximize $Z = 10x + 8y$ subject to $x + y \leq 50$; $3x + y \leq 100$; $x + 3y \leq 75$
(e) Minimize $Z = 8x + 10y$ subject to $2x + 2y \geq 100$; $2x + 6y \geq 200$; $12x + 4y \geq 300$

Exercise 6

We use the simplex method to solve the following linear programming problem:

Maximize $W = x + y - z + t$ subject to $\begin{cases} x + z - t \leq 10 \\ x + y + t \leq 20 \\ x + y - z + t \leq 30 \\ x, y, z, t \geq 0 \end{cases}$

Find the maximum value of W.

Answer: **(a)** 17 **(b)** 18 **(c)** 19 **(d)** 20 **(e)** 21

Exercise 7

What nominal rate of interest, compounded monthly, corresponds to an effective rate of 4.5%?

Answer: **(a)** 4.41% **(b)** 4.50% **(c)** 5.40% **(d)** 5.41% **(e)** 5.51%

Exercise 8

A debt of 7000 SR due in five years is to be repaid by a payment of 3000 SR now and a second payment at the end of five years. How much should the second payment be if the interest rate is 8% compounded monthly?

Answer: **(a)** 2350 **(b)** 4000 **(c)** 3250 **(d)**3520 **(e)** 2530

Exercise 9

What interest rate compounded continuously is equivalent to an interest rate of 6% compounded semiannually?

Answer: **(a)** 5.5% **(b)** 5.7% **(c)** 5.9% **(d)** 6.0% **(e)** 6.1%

Exercise 10

In 10 years, a 40,000 SR machine will have a salvage value of 4,000 SR. A new machine at that time is expected to sell for 52,000 SR. In order to provide funds for the difference between the replacement cost and the salvage value, a sinking fund is set up into which equal payments are placed at the end of each year. If the fund earns 7% compounded annually, how much should each payment be?

Answer: **(a)** 6834 SR **(b)** 2605 SR **(c)** 868 SR **(d)** 3474 SR **(e)** 5125 SR

[From Appendix A: $a_{9|0.07} = 6.515232$; $a_{10|0.07} = 7.023582$; $a_{11|0.07} = 7.498674$
$s_{9|0.07} = 11.977989$; $s_{10|0.07} = 13.816448$; $s_{11|0.07} = 15.783599$]

Exercise 11

Suppose 50 SR is placed in a savings account at the end of each month for 4 years. If no further deposits are made, how much is in the account after 6 years? Assume that the savings account pays 6% compounded monthly.

Answer: **(a)** 3048 SR **(b)** 2704 SR **(c)** 2129 SR **(d)** 12828 SR **(e)** 2399 SR

[From Appendix A: $a_{4|0.005} = 3.950496$; $a_{48|0.06} = 15.650027$; $a_{48|0.005} = 42.580318$
$s_{4|0.005} = 4.030100$; $s_{48|0.06} = 256.564529$; $s_{48|0.005} = 54.097832$]

Exercise 12

When at least one of four flags colored red, green, yellow, and blue is arranged vertically on a flagpole, the result indicates a signal (or message). Different arrangements give different signals. How many different signals are possible if at least one flag is used?

Answer: **(a)** 15 **(b)** 64 **(c)** 4 **(d)** 96 **(e)** 79

Exercise 13

A waiter takes the following order from a table with seven people: 3 hamburgers, 2 cheeseburgers, and 2 steak sandwiches. Upon returning with the food, he forgets who ordered what item and simply places an item in front of each person. In how many ways can the waiter do this?

Answer: **(a)** 25200 **(b)** 24 **(c)** 1050 **(d)** 12 **(e)** 210

Exercise 14

A stock is selected at random from a list of 60 utility stocks, 48 of which have an annual dividend yield of 6% or more. Find the probability that the stock pays an annual dividend that yields less than 6%.

Answer: **(a)** 0.1 **(b)** 0.2 **(c)** 0.3 **(d)** 0.4 **(e)** 0.5

Exercise 15

If P(E) = 1/4, P(E∪F) = 7/12, and P(E∩F) = 1/6, find P(E|F'). [Hint: Use the identity P(E) = P(E∩F) + P(E∩F')]

Answer: **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{2}{3}$ **(e)** $\frac{5}{6}$

Exercise 16

In a certain town, 40% of eligible voters are Democrats, 35% are Republicans, and the remainder are Independents. In the last presidential election, 15% of the Democrats, 20% of the Republicans, and 10% of the Independents voted. If an eligible voter is chosen at random, what is the probability that he or she voted?

Answer: **(a)** 0.90 **(b)** 0.85 **(c)** 0.35 **(d)** 0.15 **(e)** 0.10

Exercise 17

A first card is drawn from a deck of 52 cards. Then a second card is drawn.
Let E={First card is a Heart}, F={First card is a Diamond}, and G={Second card is a Heart}. **Then:**

(a) G and E are independent **(b)** P(G) - P(F) = P(E) **(c)** P(E) + P(F) = 2P(G) **(d)** P(G | E) = $\frac{12}{52}$ **(e)** P(G|F) = $\frac{12}{51}$

Exercise 18

The monthly salaries of the employees of a company are 300 SR, 500 SR, 1,200 SR, 1,500 SR, 3,000 SR, 10,000 SR with corresponding frequencies 8, 5, 3, 2, 1, 1, respectively. How many standard deviations from the mean is the highest salary?

Answer: **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

Exercise 19

A jar contains two red and three white marbles. Two marbles are randomly withdrawn in succession with replacement. Let X be the number of red marbles withdrawn. Find $Var(X)$.

Answer: **(a)** $\frac{8}{25}$ **(b)** $\frac{9}{25}$ **(c)** $\frac{10}{25}$ **(d)** $\frac{11}{25}$ **(e)** $\frac{12}{25}$

Exercise 20

In a game, a coin is tossed 3 times. You gain 100 SR if either 1 head or 2 heads turn up. You lose 300 SR in the other cases. Find your expected gain in this game.

Answer: **(a)** -200 SR **(b)** -150 SR **(c)** 0 SR **(d)** 150 SR **(e)** 200 SR

Exercise 21
In a production process, the probability of a defective unit is 0.01. A sample of 1,000 units is selected at random. Find the probability that 999 units are NOT defective.

Answer: **(a)** 0.4 **(b)** 0.04 **(c)** 0.004 **(d)** 0.0004 **(e)** 0.00004

Exercise 22
From a deck of 52 cards, 8 cards are randomly drawn in succession with replacement. What is the probability that there are exactly 2 spades?

Answer: **(a)** $\frac{(3^2)(7)}{2^{14}}$ **(b)** $\frac{(3^2)(14)}{2^{14}}$ **(c)** $\frac{(3^4)(14)}{2^{14}}$ **(d)** $\frac{(3^6)(7)}{2^{14}}$ **(e)** $\frac{(3^6)(14)}{2^{14}}$

Exercise 23
Suppose X is a binomially distributed random variable with $\mu = \frac{3}{5}$ and $\sigma^2 = \frac{12}{25}$. Find $P(X = 2\ or\ 3)$.

Answer: **(a)** 0.008 **(b)** 0.096 **(c)** 0.104 **(d)** 0.896 **(e)** 0.904

Exercise 24
Assume that Z is a standard normal random variable. Find z_0 such that $P(|Z| > z_0) = 0.3174$.

Answer: **(a)** 1 **(b)** 1.5 **(c)** 2 **(d)** 2.5 **(e)** 3

[From Appendix C: $A(1) = 0.3413$; $A(1.5) = 0.4332$; $A(2) = 0.4772$; $A(2.5) = 0.4938$; $A(3) = 0.4987$]

Exercise 25
The scores on a national achievement test are normally distributed with mean 500 and standard deviation 100. What percentage of those who took the test had a score less than 700?

Answer: **(a)** 2.28% **(b)** 7.44% **(c)** 47.72% **(d)** 95.44% **(e)** 97.72%

[From Appendix C: $A(1) = 0.3413$; $A(1.5) = 0.4332$; $A(2) = 0.4772$; $A(2.5) = 0.4938$; $A(3) = 0.4987$]