1. The length of the curve $y = 1 + \frac{2}{3}x^{3/2}, \quad 0 \le x \le 3$, is

(a) $\frac{14}{3}$

(b) $\frac{13}{17}$

(c) $\frac{7}{15}$

(d) $\frac{12}{7}$

(e) $\frac{11}{6}$

2. The area of the surface generated by revolving $y = 2\sqrt{1-x}$ between $x = -1$ and $x = 0$ about the x-axis is equal to

(a) $\frac{8\pi}{3}\left(3\sqrt{3} - 2\sqrt{2}\right)$

(b) $\frac{\pi}{5}\left(5\sqrt{5} + 1\right)$

(c) $\frac{\pi}{2}\left(2\sqrt{2} + 2\right)$

(d) $\pi(\sqrt{7} + 1)$

(e) $\frac{\pi}{3}\left(3\sqrt{3} + \sqrt{5}\right)$

3. The sequence $\left\{\dfrac{\sin^2 n}{2^n}\right\}_{n=1}^{\infty}$

(a) converges to 0

(b) converges to 1

(c) converges to 2

(d) converges to 4

(e) diverges

4. $\displaystyle\sum_{n=0}^{\infty} \frac{3(-2)^n}{8^n} =$

(a) 2.4

(b) 1.3

(c) 1

(d) 7.8

(e) 11.2

5. $\sum_{n=2}^{\infty} \frac{2}{n^2-1} =$

(a) $\frac{3}{2}$

(b) $\frac{1}{2}$

(c) $\frac{5}{2}$

(d) 0

(e) 1

6. The series $\sum_{n=1}^{\infty} \frac{1}{n^{p^3-7}}$ is convergent if p belongs to

(a) $(2, \infty)$

(b) $(-\infty, 2)$

(c) $(-7, 0)$

(d) $(-\infty, -7)$

(e) $(-\infty, 1)$

7. The series $\sum_{n=1}^{\infty}\left(\frac{1}{n^3}\right)$ is convergent by the integral test. If its sum is approximated by the sum of the first two terms then the remainder R_2 belongs to the interval

(a) $\left[\frac{1}{18},\frac{1}{8}\right]$

(b) $\left[\frac{1}{9},\frac{1}{6}\right]$

(c) $\left[\frac{1}{4},\frac{1}{2}\right]$

(d) $\left[0,\frac{1}{8}\right]$

(e) $\left[\frac{1}{36},\frac{1}{8}\right]$

8. The series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[5]{n^2}}$ is

(a) conditionally convergent

(b) absolutely convergent

(c) divergent by integral test

(d) divergent by divergence test

(e) convergent by integral test

9. The series $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+1}\right)^{3n}$ is

(a) convergent by the root test

(b) divergent by the root test

(c) divergent by the integral test

(d) convergent by the integral test

(e) divergent by the ratio test

10. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{2n+1}$ is

(a) $(2, 4]$

(b) $[2, 4)$

(c) $(4, 6]$

(d) $[-1, 1)$

(e) $[4, 6)$

11. The sum of the first three terms of Taylor series representation of the function $f(x) = \ln(1 + x^2)$ about $a = 2$ is

(a) $\ln 5 + \frac{4}{5}(x-2) - \frac{3}{25}(x-2)^2$

(b) $\ln 5 + \frac{1}{5}(x-2) - \frac{3}{7}(x-2)^2$

(c) $\ln 5 + \frac{4}{5}(x-2) - \frac{7}{35}(x-2)^2$

(d) $2\ln 5 + \frac{9}{7}(x-2) - \frac{13}{23}(x-2)^2$

(e) $\ln 5 + \frac{3}{5}(x-2) - \frac{9}{21}(x-2)^2$

12. If $s_n = a_1 + a_2 + \cdots + a_n = \ln\left(2n^2 - 1\right) - \ln\left(n^2 + 1\right)$, then $\sum_{n=1}^{\infty} a_n =$

(a) $\ln 2$

(b) e^2

(c) $\ln 1/2$

(d) $e^{1/2}$

(e) 0

13. In the interval $(-\sqrt[3]{3}, \sqrt[3]{3})$, the power series representation of the function $f(x) = \dfrac{1}{x^3+3} =$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{3^{n+1}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3^n}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{3n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^{n+1}}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n-3}}{2^n}$

14. Which of the following statement is true about the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+5}}{2n^2+n+3}$?

(a) It converges by the limit comparison test

(b) The limit comparison test is inconclusive

(c) It is telescoping series and its sum is $\dfrac{1}{2}$

(d) It diverges by the divergence test

(e) It diverges by the limit comparison test

15. $\int \dfrac{4(x^3+1)}{x^2-4}dx =$

(a) $2x^2 + 9\ln|x-2| + 7\ln|x+2| + C$

(b) $2x^2 + 5\ln|x-2| + 3\ln|x+2| + C$

(c) $2x^2 + \ln|x-2| + 6\ln|x+2| + C$

(d) $3x^2 + 8\ln|x-2| + 2\ln|x+2| + C$

(e) $3x^2 + 3\ln|x-2| + 12\ln|x+2| + C$

16. The improper integral $\int_0^1 \dfrac{x+1}{\sqrt{x^2+2x}}\,dx$

(a) converges to $\sqrt{3}$

(b) converges to 1

(c) converges to $\sqrt{2}$

(d) converges to 4

(e) diverges

17. $\displaystyle\int_{\ln 2}^{\ln 3} \frac{dx}{e^x - 1} =$

(a) $\ln\left(\frac{4}{3}\right)$

(b) $\ln\left(\frac{3}{7}\right)$

(c) e^5

(d) 1

(e) $\ln 6$

18. $\displaystyle\int_{1/2}^{1} \frac{5(1-x^2)^{3/2}}{x^6}\,dx =$

(a) $9\sqrt{3}$

(b) $8\sqrt{3}$

(c) $3\sqrt{3}$

(d) $10\sqrt{3}$

(e) $27\sqrt{3}$

19. $\int_0^1 2x \tan^{-1} x \, dx =$

(a) $\dfrac{\pi - 2}{2}$

(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi - 1}{4}$

(d) $\dfrac{\pi}{8}$

(e) $\dfrac{1 + \pi}{8}$

20. The slope of the tangent line to the curve $y = \int_0^{x^4} \dfrac{t^2}{t^2 + t + 2} \, dx$ at $x = 1$ is

(a) 1

(b) -1

(c) 0

(d) 2

(e) -2

21. $\int_0^2 |2x-1|\,dx =$

(a) $\frac{5}{2}$

(b) $\frac{1}{2}$

(c) 2

(d) 3

(e) 0

22. Using the method of cylindrical shells, the volume of the solid obtained by rotating the region enclosed by $y = x^3$, $y = 1$ and $x = 0$ about $y = -3$ is given by the integral

(a) $2\pi \int_0^1 (3+y)(\sqrt[3]{y})\,dy$

(b) $2\pi \int_0^1 (y-3)(y^3)\,dy$

(c) $\pi \int_{-3}^0 (1+y)(\sqrt[3]{y})\,dy$

(d) $\pi \int_0^1 (x^3-3)^2\,dx$

(e) $2\pi \int_0^1 \left[(x-3)^2 - x^3\right]\,dx$

23. Let c be a positive real number, such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 9, then $c =$

(a) $\frac{3}{2}$

(b) $\frac{1}{3}$

(c) $\frac{2}{5}$

(d) 3

(e) 4

24. The average value of the function f, whose graph is given below, on the interval $[0, 6]$ is

(a) $\frac{11}{12}$

(b) $\frac{13}{12}$

(c) $\frac{15}{12}$

(d) $\frac{7}{12}$

(e) $\frac{5}{12}$

25. $\int \tan^4 x\, dx =$

(a) $\frac{1}{3}\tan^3 x - \tan x + x + C$

(b) $\frac{1}{2}\sec^3 x - x\sec x + C$

(c) $\frac{1}{3}\tan^3 x + \sec^3 x + C$

(d) $\frac{1}{5}\tan^5 x - \sin x + x^2 + C$

(e) $2\sec^3 x + 3\tan^3 x + 3x^2 + C$

26. $\int \sin^4 x \cos^5 x\, dx =$

(a) $\frac{\sin^9 x}{9} - \frac{2\sin^7 x}{7} + \frac{\sin^5 x}{5} + C$

(b) $\frac{\sin^7 x}{7} - \frac{2\sin^5 x}{5} + \frac{\sin^3 x}{3} + C$

(c) $\frac{\cos^9 x}{7} - \frac{3\cos^7 x}{14} + \frac{\cos^3 x}{5} + C$

(d) $\frac{\cos^9 x}{7} + \frac{\cos^7 x}{14} + \frac{\cos^3 x}{5} + C$

(e) $\frac{\cos^9 x}{9} + \frac{5\sin^7 x}{7} + \frac{\cos^3 x}{3} + C$

27. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!} =$

(a) 0.5

(b) 1

(c) 0.3

(d) ln 2

(e) 0.25

28. $\displaystyle\lim_{x\to 0} \frac{x^3 - 3x + 3\tan^{-1} x}{6x^5} =$
(**Hint: You may use power series)**

(a) 0.1

(b) 0.2

(c) 0.3

(d) 0.4

(e) 0.5