SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

(1) (5 points) Find the absolute minimum values of the function $f(x) = \dfrac{\ln x}{x^2}$ on the interval $[1, e]$.

(2) (5 points) Verify that the function $f(x) = x^3 - 4x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1,3]$. Find all values of c that satisfy the conclusion of the theorem.

(3) (10 points) Evaluate the limit if it exists:

   a. $\lim_{x \to 1^+} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

   b. $\lim_{x \to \frac{\pi}{2}^-} (\sec x)^{\cos x}$

(4) (20 points) Given the function

$$y = f(x) = \frac{x^2}{x+1} \quad \text{with} \quad f'(x) = \frac{x(x+2)}{(x+1)^2} \quad \text{and} \quad f''(x) = \frac{2}{(x+1)^3}$$

a. (2 Points) Find the asymptotes if any exist.

   Horizontal:
   Vertical:
   Slant:

b. (2 Points) Find the critical numbers.

c. (2 Points) Find intervals where the function is increasing and those where it is decreasing.

d. (2 Points) Find the local maximum and minimum of the function.

e. (3 Points) Discuss the concavity of the function and find the infection points.

f. (9 Points) Sketch the graph of the function. Clearly indicate the **critical numbers**, **extrema** and **inflection points** on the graph.